**A matching model of R&D cooperation**

by

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## Biographic note

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While studying at FEP.UP, and since the beginning of 2012, Catarina started to collaborate with Rede Ambiente and Vector Estratégico in the elaboration of economic studies for public tenders. Since 2014, Catarina has been working at Sonae in the Investor Relations department

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**Resumo:** Esta dissertação tem como objetivo o desenvolvimento de um modelo de *matching* de cooperação em I&D no qual as empresas tomam decisões de aliança baseadas no *fit* tecnológico dos potenciais parceiros. Em particular, pretende-se estender a literatura existente sobre cooperação em I&D avaliando se as características tecnológicas das empresas (por exemplo, a distância tecnológica, investimento em I&D) influenciam a formação da rede. Após o desenvolvimento de um modelo de *matching* de cooperação em I&D, resolvemos o jogo considerando um cenário de duopólio com empresas homogéneas. Depois, consideramos um cenário com *n* empresas heterogéneas e com *t* períodos que foi analisado através de um modelo de simulação multi-agentes. Os resultados mostram que a distância tecnológica das empresas é positivamente correlacionada com o investimento em I&D, com o número de ligações e com a centralidade das empresas na rede. Adicionalmente, o investimento em I&D é positivamente correlacionado com a com a centralidade das empresas (*Degree*, *Betweenness* ou *Closeness)*, assim como, com o lucro e a quantidade produzida pela empresa.

Códigos JEL: C78, D40, O30.

Palavras-chaves: Cooperação em I&D; *matching*; modelos multi-agentes; simulação.

**Abstract:** This research aims at developing a matching model of R&D cooperation in which firms endowed with knowledge elements make their alliance decisions based on the technological fit of potential partners. Particularly, we intend to extend existing literature on R&D cooperation by evaluating if firms' technological characteristics (e.g. technological distance, R&D investment) influence the network formation. After the development of a matching model of R&D cooperation, we solve the game considering a duopoly scenario with homogeneous firms. We then consider a set-up with *n* heterogeneous firms and *t* periods that was analysed through agent-based modelling. Our results show that firms' technological distance is positively correlated with R&D investment and with the number of links and firms' centrality within the network. In addition, the R&D investment is positively correlated firm's centrality (*Degree, Betweenness* or *Closeness)* as well as with the firm's profit and output.

JEL Codes: C78, D40, O30.

Key words: R&D cooperation; matching; agent-based modelling; simulation.

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# Chapter 1 - Introduction

Research and Development (henceforward R&D) activities are rather different from conventional production activities because knowledge has the attributes of a public good, and therefore, a rival can appropriate the research of another firm due to the existence of knowledge spillovers. In fact, the existence of R&D spillovers cause insufficient private incentives to invest in R&D and can make the competitor stronger even without any effort (Katz, 1986). Cooperation between firms allows them to internalise the benefits of research, as well as to capture economies of scale and complementarities in R&D, and may be a solution to this problem. Against these advantages is the risk that cooperating firms may free-ride on other firms, as well as the possibility of reduction of competition in the product market, which would result in a welfare loss.

Firms share some characteristics (like technology, product, etc.) that allow them to reach benefits when they cooperate in R&D. In particular, and according to Kamien *et al*. (1992), firms can achieve a competitive-advantage externality and a combined-profits externality. This externalities cause a positive effect when the spillover rate is sufficiently high. As is pointed by the pioneering work of d'Aspremont and Jacquemin (1988), R&D cooperation may lead to adjustments in the output, in the unit production cost and in the R&D costs themselves. This means that changes related to how R&D is done can really affect key areas of a firm. Thereby, firms can achieve major benefits if they cooperate with each other on R&D leading to higher levels of both producer and consumer surplus.

R&D cooperation may be structured according to a network. A network is characterised by a set of links between two or more firms. Therefore, to develop those links, one firm must have something that the other one wants, in order to improve their performance (Goyal and Moraga-González, 2001).

In a network, firms can join efforts and gain a competitive advantage against their rivals that would not be possible if they were working by themselves (Zirulia, 2011). It is important to underline that firms can relate not only vertically but also horizontally. In fact, and as is pointed by Goyal and Moraga-González (2001), the proportion of collaborations among firms that have some degree of rivalry, or horizontal collaborations, is quite significant.

Matching theory helps to understand how a network is formed. Matching consists in the correspondence of one member of a group to one or more members of another group that somehow have some interest in being together (Roth, 1982). A matching model has the objective of do the best match possible which makes both parts fulfilled of being together and unwilling of change it. According to Roth (1982), matching may range from a completely decentralised procedure, in which agents negotiate directly with one another (as in marriage in contemporary Western societies), to a completely centralised procedure, in which all agents state their preferences for possible matches, which are then assigned according to some specified algorithm (as in the procedure by which students are matched with the universities in which they want to graduate). Matching models have been applied to different topics: students with educational institutions, job market for graduating medical students, athletes with teams, and men with women. Recently, matching has been employed in the study of R&D networks (e.g. Li-ping, 2006; Cowan *et al*., 2009; Santamaria and Surroca, 2011).

The main objective of this research is the development of a matching model of R&D cooperation in which firms make their alliance decisions based on the technological fit of potential partners. In particular, this work intends to analyse the influence of firms' technological characteristics (e.g. technological distance, R&D investment) on the formation of R&D networks. This topic is particularly relevant because it helps to understand first, how firms choose their partners for R&D purposes, and second, how networks change with technological variables.

This research contributes to the scientific area where it fits because it joins the topics of R&D cooperation games, matching theory and agent-based modelling. In spite of a significant proliferation of research on matching models and on R&D cooperation, their combination is quite sparse making this theme so innovative. Furthermore, this work is grounded in existing literature of Goyal and Moraga-González (2001), Cowan *et al*. (2009) and Campos *et al*. (2013) and extends it, by focusing on the influence of firms' technological profiles on the formation of R&D cooperation networks.

Therefore, this dissertation aims to answer the following questions: Are R&D cooperation networks influenced by the technological characteristics of firms (R&D investment, technological distance)? Who are the most central firms in the network and how is firms' centrality influenced by technological variables?

From the methodological point of view, this research starts by developing a matching model of R&D cooperation. We then present an analytical resolution of the game considering a duopoly scenario with homogeneous firms. After that, we consider a set-up with a large number of heterogeneous firms with respect to technology, as well as several decision periods. The resulting complexity of the model calls for the development of an agent-based model allowing us to explore the evolution of firms' alliances for R&D purposes by means of simulation.

This work is structured as follows: after this Introduction the text presents a literature review on the topics of R&D cooperation in chapter 2 and matching models in chapter 3. In Chapter 4 it is developed a matching model of R&D cooperation through game theory modelling which is analytically solved for the duopolistic case. This chapter also includes an Agent-Based model that allows simulating the results of matching models and ends with a statistical analysis of the results of the simulation. Finally, in the last section it is presented the main conclusions of the dissertation and future developments.

# Chapter 2 - R&D cooperation models: an overview

Following Falvey *et al*. (2013), firms have two main incentives to invest in R&D: the reduction of their own production costs and the creation of a cost gap between itself and its rivals. According to d'Aspremont and Jacquemin (1988), when a firm does R&D, some benefits of that research flows to other firms without payment, a phenomenon called spillover. This happens due to the public nature of knowledge. If spillovers did not exist, all the benefits would be appropriated by the firm who developed R&D and their rivals would not withdraw any advantages of that research (Falvey *et al*., 2013).

Firms can cooperate with each other in R&D activities and/or they can extend their cooperation and be partners at the production level. Cooperating in R&D may bring some benefits to the participating firms. Authors like Kamien *et al*. (1992) and Grossman and Shapiro (1986) identify the following benefits from the R&D cooperation: eliminate the duplication of effort; overcome the free rider problem; promote the diffusion of research findings; allow the participants to enjoy economies of joint research (cross-firm synergies); and enable the participants to overcome the cost of research and development that is a barrier to a firm alone. Despite all of that, R&D cooperation has some disadvantages that may diminish the incentives to do research and therefore, may reduce the social welfare. The same authors identify curtail competition in other phases of their interaction and danger of collusion in the product market as possible weaknesses of R&D cooperation.

According to Grossman and Shapiro (1986) the majority of the output produced by a Research Joint Venture (henceforward RJV) is information and like so it is essential a soft antitrust treatment different from the others ventures like production or marketing ventures. Firms who develop R&D activities are, in some way, protected by a patent system but, even with property rights, the problem of appropriability of information by other firms is not completely solved (Grossman and Shapiro, 1986). In fact, and as is mentioned by these scholars, there are spillovers which are not prevented by patents and this can lead to underinvestment in R&D. Their second point is that knowledge has the attributes of a public good meaning that the more widespread the information, the best the economic efficiency. The authors conclude that RJV can be a solution for these two problems in many occasions. In a RJV, firms share the R&D costs, solving, in part, the first problem. They also share the R&D results, solving, in some measure, the second problem since the information is automatically widespread between the firms. When a RJV is formed it is possible that patents are lower for non-members and that leads to an increase downstream production costs (in case of process innovation) or to restrict downstream competition (for product innovation) (Grossman and Shapiro, 1986).

To better understand this issue many authors formalised R&D cooperation through models that are generally structured as a two-stage oligopoly game where, in the first stage, firms choose their R&D level of expenditure or output and, in the second stage, firms decide their output (Cournot competition) or price (Bertrand competition) in order to maximize joint profits. R&D cooperation models can differ in the number of participants; type of product (homogeneous or differentiated) and spillover (exogenous or endogenous), among other specificities. In annex 1 it is presented a table with several articles on R&D cooperation and their main characteristics and conclusions.

One of the first authors to study this subject was Katz (1986). He analysed a four stage game where firms decide, in the first stage, if they should participate or not in a cooperative research agreement; in the second stage, they decide the R&D cost-sharing and R&D output-sharing rules; in the third stage and given the decisions of the previous stage, each firm chooses its level of R&D effort; in the last stage, firms choose their levels of output. He concludes that firms that share R&D costs have more incentives to develop R&D and firms who share R&D output are more efficiency on R&D. Another interesting conclusion is that the probability of an industry wide agreement that increases effective R&D is smaller when the competition between firms is larger. This happens because part of the benefits accomplished with the reduction in unit costs goes to the consumer instead of the producer. Another important conclusion is that industries with high spillovers have more incentives to develop R&D cooperation.

d'Aspremont and Jacquemin (1988) also studied this theme but considered cooperation only between two firms in a two-stage model, homogeneous products and a quadratic cost function. In the first stage firms decide their R&D expenditure levels and in the second the output. The authors conclude that firms that cooperate in R&D and are rivals in the product market can achieve higher levels of R&D and can increase the quantity produced for large spillovers ($\beta$>0.5), comparatively to firms that have not any kind of cooperation. When firms act as a monopoly and cooperate in both R&D and output, they have higher levels of R&D for high spillovers in comparison to the fully non-cooperative equilibrium and to the pure R&D cooperation equilibrium. The main conclusion is that cooperative behaviour can be positive in a small industry where R&D generates spillovers.

Other authors that study this issue are Kamien *et al*. (1992). Although they define a two stage model where in the first stage firms invest in R&D and in the second stage they choose the price (Bertrand) or the quantity produced (Cournot), their model is different from d'Aspremont and Jacquemin (1988), because they considered $n$ firms, a general concave R&D production function, differentiated products and a different spillover effect between cooperating and non-cooperating firms. They conclude that a RJV that cooperates in its R&D expenditure decisions yields the highest consumer plus producer surplus under Cournot competition and, in most cases, under Bertrand competition.

Suzumura (1992) also examine the positive and normative effects of cooperative R&D in comparison with non-cooperative R&D. For that purpose he considers $n$ participants under Cournot competition, producing a homogeneous output and a two-stage model where in the first stage firms decide on their cost-reducing R&D under competition or cooperation and in the second stage firms face quantity competition in the product market. His main conclusions are that in the presence of sufficiently large R&D spillovers neither non-cooperative nor cooperative equilibria achieve even second-best R&D levels but, in the absence of spillover effects, the cooperative R&D level remains socially insufficient and the non-cooperative level may overshoot first and second best levels of R&D.

Another scholar who also studies this issue is Poyago-Theotoky (1995). She considers an oligopoly model with information spillovers and intends to understand, first, how firms outside the RJV are affected by the RJV formation and second, if it is cheaper to develop R&D activities outside or inside the RJV. When firms act non-cooperatively, they achieve higher cost reduction inside the RJV; in most of the cases of the RJV scenario, firms inside the RJV achieve better profits than the outsiders. Another main conclusion of her study is that the market may not provide enough incentives for the optimum size of the RJV, for some levels of information spillovers.

Matsumura *et al*. (2013), study the relation between the degree of competitiveness faced by firms and their R&D expenditure. In their study the authors consider a two-stage symmetric duopoly model for homogeneous products and under Cournot competition. In this model, Matsumura *et al*. (2013) assume that the payoff of a firm depends on absolute and relative profits. In the first stage firms choose their R&D level whereas, in the second stage and after observing the rivals R&D level, firms produce goods that are perfect substitutes. These authors conclude that the relation between the degree of competitiveness and the level of innovation activities is U-shaped which means that R&D activities are larger in highly cooperative (monopoly markets) and non-cooperative (perfectly competitive markets) industries and have less incidence in intermediate cases (duopoly markets). Matsumura *et al*. (2013) also study the joint R&D implementation in which firms cooperate in the first stage choosing the R&D level in order to maximise their joint profits while they still rivals at the output level. In this case the authors conclude that there is a monotone relation between R&D level and competitive level, unlike the case of non-cooperation in R&D. Their main conclusion, in this case, is that an increase in the degree of competitiveness leads to a reduction of R&D activities.

The above mentioned authors assumed that R&D spillovers were exogenous, that is, the level of information disclosure from one firm to other is not chosen by firms. However, some scholars consider endogenous R&D spillovers. Among these are Katsoulacos and Ulph (1998), who formalised a three-stage model with two firms. In the first stage firms decide a line of research to pursue. All lines allow firms to reach the same unit progress if it succeeds in making a discovery; however, they differ on the capacity of the other firm to adapt the discovery to its own use. Firms decide the amount of R&D in the second stage and, if the discovery reaches success, the fraction of the information the firm wishes to share with the other firm in the third stage. The main conclusion of this work is that RJVs may sometimes act in an anti-competitive way when some features occur. This behaviour is reflected by choosing partial RJV spillovers or by closing a R&D lab.

Kultti and Takalo (1998) also consider endogenous R&D spillovers. They developed a three-stage game where, in the first stage, firms invest in cost reducing R&D; in the second stage firms play a game where they may exchange their R&D results if both agree; in the last stage, firms play the Cournot game. They conclude that the spillover of R&D can be endogenised in a sense that even without spillovers firms have an incentive to exchange the R&D information after the investment costs are sunk. Another conclusion is that firms always exchange information if the degree of spillover is symmetric.

Similarly, Poyago-Theotoky (1999) considers Cournot competition among two firms that produce a homogeneous product and analyses a non-tournament model of R&D where firms are engaged in cost-reducing innovation. She defines a three-stage game: in the first stage, firms decide their cost-reducing R&D expenditure, while in the second stage the information spillovers turns to be endogenous by making firms decide how much of the knowledge created in the first stage will be disclosed; in the last stage, firms compete in the product market. Her main conclusions are that when firms choose their R&D non-cooperatively they never disclose any of the information, while under cooperative R&D, firms will always choose to fully share their information.

Some authors formalised models where the size of the cooperative research identity is endogenous. Combs (1993) consider that firms can choose the size of the RJV that is more beneficial for the participating firms. In order to analyse if R&D cooperation allows firms to increase the probability of discovering a new product Combs (1993) develop a three stage model where in the first stage each firm decide to joint, or not, the RVJ; in the second stage, each participating firm decide their R&D expenditure which maximise collective expect net returns and non-cooperating firms choose their own R&D expenditure level each maximise their individual expect net returns; in the last stage, firms find out if their research was successful and if so enter the output market competing with each other. In his model Combs (1993) also considers homogeneous products, $n$ participants and a symmetric Cournot-Nash production game. To accomplish results the author use simulation. One of his main conclusions is that when there is a huge probability of success, more firms decide to cooperate, expected consumer surplus increases but expected net returns to R&D of the industry decreases. This happens due to the increase in the probability of reach innovation that leads to a fierce product market competition. Despite these contradictory positions, welfare increases. Another main conclusion of Combs (1993) is that when the probability of reach success is low, firms do not form RJV because it does not bring benefit for the participating firms and is socially undesirable. Combs (1993) conclude that the number of cooperating firms never exceeds the welfare optimum. Moreover, when comparing RJV (with restrictions) and pure competition, the welfare in the first situation is higher.

Other authors focus on certain specificities or asymmetries in R&D spillovers. One of those authors is Vonortas (1994) who considers diverse degrees of spillover between cooperating firms taking into account the type of research (generic or specific). He considers a three-stage model with two firms producing homogeneous goods. In the first stage firms decide to undertake generic research; in the second stage firms choose the development research and in the last stage firms face Cournot competition. He conclude that joint ventures that simply allow members to coordinate their actions in pre-competitive research can restore firm incentives for both pre-competitive R&D in the presence of high knowledge spillovers and poor opportunities for innovation; joint ventures that additionally improve the dissemination of information among member firms raise social benefits whenever the opportunities for innovation are not exceptionally good, even in the presence of relatively insignificant spillovers.

Steurs (1995) extends literature to a two industry each one with two firms. Doing this he made possible for R&D spillovers occur within and between industries. In Steurs (1995) two-stage model, firms choose their level of R&D investments in the first stage and in the second firms decide the quantity to produce and sell on the market. His main conclusions are that inter-industry R&D spillovers have a very important effect on a firm’s incentives to invest in R&D and inter-industry R&D agreements may be more socially beneficial than intra-industry R&D agreements.

Amir and Wooders (1999) differ from previous literature because they consider one-way spillovers in their study turning R&D spillovers to be asymmetric. This means that know-how only flows from a R&D intensive firm (innovator) to their rival (imitator) leading to an asymmetric equilibrium. To analyse the effects of one-way spillovers Amir and Wooders (1999) consider a typical two-stage R&D-output game. Their main conclusion is that a joint lab always improves on consumer welfare, yields higher profits, cost reduction and social welfare only under extra assumptions, beyond those required with multidirectional spillovers

Another important concept in this literature is absorptive capacity. According to Kamien and Zang (2000) to realise the rival's spillovers, firms have to develop their own R&D otherwise spillovers are not useful. To understand this concept they formalised a three-stage model with two participating firms producing homogeneous output and under Cournot competition. In the first stage each firm choose its R&D type (that could be firm-specific R&D or generic R&D); in the second stage firms choose the level of investment in R&D and in the last stage firms face Cournot competition. They conclude that when firms cooperate in the setting of their R&D budgets, i.e. form a RJV, they choose identical broad R&D approaches. Furthermore, if they do not form a RJV, then they choose firm-specific R&D approaches unless there is no danger of exogenous spillovers.

Youssef *et al*. (2011) also analyse the case where firms can invest in both innovative and absorptive R&D to reduce their unit production cost considering spillovers. For that goal they consider a two-stage model with two participating firms producing homogeneous goods. In the first stage firms invest in R&D (original and absorptive research) and in the second stage firms enter in the output game. Their main conclusion is that the investment in innovative R&D is always higher than in absorptive R&D. They also conclude that the value of the learning parameter has almost no impact on innovative R&D, firms' profits, consumer's surplus and social welfare.

R&D cooperation models may be static or dynamic. The above-mentioned authors consider static models, that is, firms who participate in the game make their choices simultaneously. Petit and Tolwinski (1999) consider that firms do their choices in different time periods (dynamic model). For that goal the authors formalised a dynamic model with two firms producing homogenous goods where firms have to decide how much to produce and how much to invest in R&D. Petit and Tolwinski (1999) conclude that antitrust legislation should be flexible towards technological cooperation since it may produce social benefits and even reduce the incentives for industrial concentration. However, the private incentives of the firms to form technological cartels may change from case to case.

Most of the models analysed assume that forming or running a RJV is costless. Falvey *et al*. (2013) differ from those studies because they consider that cooperation has costs that are higher for a large number of cooperating firms. In their study they try to understand how cooperation costs influence the RJV performance. For that, the authors consider an oligopolistic environment, *n* identical firms selling homogeneous products and a two-stage game. In the first stage firms choose their R&D level and, in the second one, firms compete with each other choosing the quantity to produce. They conclude that RJV can be profitable but welfare reducing while R&D competition can generate a better outcome depending on the extent of coordination costs.

Due to the complex nature of R&D activities it is difficult to determine the optimal policy towards R&D. Leahy and Neary (1997) study this issue using a two-stage game with *n* firms. In the first period firms choose their R&D level while in the second one firms decide the level of an action (output or price). They conclude that, except when R&D spillovers are low and firms' actions are strategic substitutes, strategic behaviour by firms tends to reduce output, R&D, and welfare and so justifies higher subsidies.

In the traditional R&D cooperation literature, although it is implicit the creation of collaborative relations between firms, it does not explicitly refer to R&D networks. A network is characterised by a set of links between two or more firms. The opportunity to create these links is prior to market interaction and, in the case of R&D, the purpose of the network may be to share R&D knowledge about a cost-reducing technology (Goyal and Moraga-González, 2001).

Goyal and Moraga-González (2001) are the first authors to explicitly analyse R&D cooperation networks. They consider an oligopoly with (ex-ante) identical firms. Prior to market interaction, each firm has an opportunity to form collaborative links with other firms in order to share R&D knowledge about a cost-reducing technology. The collection of links between firms defines a collaboration network. Given that, firms choose a (costly) level of effort in R&D unilaterally, aimed at reducing production costs. Given these costs, firms operate in the market by setting quantities in independent markets, or in a homogeneous-product market. The scholars conclude that there is a difference between a situation of absence of firm rivalry and a situation where market rivalry is strong. In the first case, the complete network (where each firm collaborates with all others) is uniquely stable, industry-profit maximizing, and efficient. In the second case, the complete network is stable, but intermediate levels of collaboration and asymmetric networks are more attractive from a collective point of view. Goyal and Moraga-González (2001) main conclusion is that competing firms may have excessive incentives to form collaborative links.

Deroian and Gannon (2006) extend previous paper by considering quality-improving investments and conclude that R&D efforts decrease with the number of partners. Goyal et al. (2008) analyse the case in which firms carry out both in-house research and bilateral joint projects and find that investments in independent research and in joint research are complementary. They also observe that a hybrid form of decision where there is bilateral cooperation yields the highest level of welfare in concentrated industries. Zirulia (2011) develop a two stage R&D-output game with *n* firms in an industry. According to this author, firms can share their efforts on a bilateral basis, and this knowledge sharing is what he defines as collaboration. In addition, he assumes that the tacit nature of technological knowledge implies that the spillover rate is never perfect, and also, the spillover rate is partner-specific, in relation to firms' technological distance. He finds that firms use the network to gain a competitive advantage and create (ex-post) asymmetries, and also that the spillover rate matters in determining the network that is optimal from the social point of view.

Other scholars like Cowan *et al*. (2009), Santamaria and Surroca (2011), Miotti and Schwald (2003), Carayol and Roux (2009) and Campos *et al*. (2013) also study R&D networks through the development of matching models. We study their contributions in the next chapter.

# Chapter 3 - Matching models

## 3.1. Introduction

Classical tools in the Marshallian tradition study markets where prices adjust in a way that supply equals demand. But there are situations in which the standard market mechanism encounters problems, and there are cases where prices cannot be used to allocate resources. Roth (2008) states that to achieve efficient outcomes, markets must be thick (meaning that, there is enough potential transactions available at one time), uncongested (there is enough time for offers to be made, accepted and rejected) and safe (safe to act straightforwardly on relevant preferences). As examples of markets where prices cannot be used in the allocation process, Roth (2008) refers to the allocation of human organs transplants, student placement in schools, and the medical labour market. The matching theory could be a solution for this problem.

Matching consists in the correspondence of one member of a group of agents to one or more members of another group of agents that somehow have some interest in being together (Roth, 1982). Joining students with educational institutions, athletes with teams, adoptive children with adoptive parents, men with women (in marriage, mixed doubles or computer dating), civil servants with civil service positions and authors with scholarly journals are, for Roth (1982), some of many important examples where matching can be used. In annex 2 a summary is presented containing the characteristics and conclusions of several articles on matching models.

The matching procedure may be characterised, according to Roth (1982), as decentralised or centralised. A decentralised procedure occurs when agents negotiate directly with one another (as in marriage in contemporary Western societies). In the opposite situation, a centralised procedure, all agents state their preferences for possible matches, which are then assigned according to some specified algorithm or procedure coordinated at a higher level (as in the procedure by which students are matched with the universities in which they want to graduate).

In some cases, a centralised procedure is used to organise markets suffering from failures (congestion or the safety of revealing private information) (Niederle *et al.*, 2008). In those cases a clearinghouse is created and its purpose is to match the participating agents. Centralised procedures may, in some cases, be the best procedure to use because, according to Niederle *et al*. (2008), centralised clearinghouses can help make markets thick and uncongested and avoid unravelling.

Analysing laboratory experiments stated by Kagel and Roth (2002) (*cfr*. Niederle *et al*., 2008), is possible to say that mechanisms that produces stable matchings' have been more successful than the ones that produces unstable ones. However, producing a stable matching is not sufficient to guarantee success, it is also necessary that the participants have incentives to participate in the match.

Niederle and Roth (2003) study the effects of a centralised clearinghouse through the analysis of the entry-level market of American gastroenterologists, having a centralised match from 1986 to 1996, and a decentralised one both before and after that period. This characteristic allowed the authors to compare the matching between the period when the clearinghouse was in operation and the periods both before and after that, allowing separating the effect of the clearinghouse from other changes in the market over time. They conclude that the use of a clearinghouse promotes the mobility of gastroenterologists, otherwise they are more likely to be employed at the same hospital in which they were internal medicine residents. Niederle and Roth (2003) suggest that the clearinghouse not only coordinates the timing of appointments but also increases the scope of the market in comparison to a decentralised market with early appointments.

Ehlers and Massó (2007) applied the matching procedures identified by Roth (1982) - centralised vs. decentralised - on the study of the entry-level medical markets in the United States of America (U.S.A.). These scholars discover that in the first half of the 20$^{th}$ century the matching process was decentralised but that cause inefficiencies leading to a reorganisation of the entry-level medical markets in the U.S.A. and turning it into a centralised process. One obvious advantage of a centralised process, in comparison to a decentralised one, is that, in the second case, it is difficult for agents to communicate with all possible partners and, in that way, find out their preferences. According to the authors, and after the reorganisation, the process of matching students to a hospital was made through a clearinghouse. Therefore, participants submit their preference lists to the clearinghouse and a mechanism determined a matching for the submitted lists. In that way, the mechanism chosen determined the success of the reorganisation. For

Ehlers and Massó (2007) a stable mechanism always selects a stable matching and is preferable to unstable ones.

Alkan (1988) highlights another characteristic of matching: the quantity of agents that are going to be matched. It is possible to link one member of one group to one member of another group (matching of twosomes) – it is the case of heterosexual marriage where a man is matched to a woman. But, in some cases, there are three kinds of agents to be matched in threesomes - an example could be the match of a man with a woman and with a child (forming a family). In a more generic way, Alkan (1988) considers that matching can have k-some formations.

Similarly, other authors also study this issue and call it k-sided matching (analogous to the k-some formation of Alkan (1988)). Therefore, and according to the direction of the preferences, matching can involve one-sided matching, two-sided matching or multi-sided matching. In the first case the problem involves the match of a set of objects to an agent having preferences over objects (Zhou, 1990). Some examples of one-sided matching could be the housing market (Shapley and Scarf, 1974), house allocation (Hylland and Zeckhauser, 1979), house allocation with existing tenants (Abdulkadiroğlu and Sönmez, 1999), school choice (Abdulkadiroğlu and Sönmez, 2003) or kidney exchange (Roth *et al*., 2005) (*cfr.* Pais, 2013). On the other hand, two-sided matching assigns two sides where both have preferences. Some examples include the matching of firms with workers (Sönmez, 1996), college admissions (Gale and Shapley, 1962) or, again, marriage model (Roth, 1982). The most typical cases of two sided matching are many-to-one matching where, normally, one side consists of an institution and the other side consists of individuals (Sönmez, 1996) - one college admits many students, one hospital employs many interns. If more than two agents of different skills have to be matched in order to realise a value of a transaction then this is called multi-sided matching. Sherstyuk (1999), in her work about multi-sided matching games with complementarities, gives some examples: to sell a property it is needed a buyer, a seller and a lawyer; to build a house it is necessary a future home owner, an architect and a worker; to start a new firm it is needed a capitalist, an entrepreneur and a worker.

Matching models may also have a quota restriction. According to Femenia *et al*. (2011), a quota is the maximum number of individuals that an institution can match. This happens due to the existence of more pairs of candidates than positions to be filled by the institution (q quota). The same authors justify this limitation with technological, legal or budgetary reasons.

After offering an overview of matching models, the next section presents a sketch of the main characteristics of two sided matching models, which are the most frequent ones in the literature and are the starting point of our model.

### 3.2. Main characteristics of a two-sided matching model

Two-sided matching problems are quite common in real life. In order to better understand two-sided matching problems, it will be analysed below the marriage problem which is the simplest case of this type of matching.

Studying matching implies, first of all, to define the problem. For that, it is essential to identify the sets of agents that are going to be matched and their preferences profile (rank order list). After that, it is important to develop an algorithm/mechanism in order to achieve the outcome of the game (the matching).

When considering the marriage problem (Roth and Sotomayor, 1990), matching can be described as a triple (M, W, P) where:

M = $\{m_1, \ldots, m_p\}$ is the set of men;

W = $\{w_1, \ldots, w_p\}$ is the set of women;

P = $\{P(m_1), \ldots, P(m_p), P(w_1), \ldots, P(w_p)\}$ is the set of preference list.

Thus, P($m$) is the ordered list of preferences of each man ($m$) on the set $W \cup \{m\}$ and P($w$) is the ordered list of preferences of each woman ($w$) on the set $M \cup \{w\}$.

A matching is the outcome of the marriage market:

$$\mu: M \cup W \rightarrow M \cup W$$

such that $\mu(m) = w$ iff $\mu(w) = m$ and for all $m$ and $w$, $\mu(m) \in W \cup \{m\}$ and $\mu(w) \in M \cup \{w\}$.

This means that matching decisions must be bilateral, being $w$ the solution for $m$ only if $m$ is the solution for $w$ otherwise one of them will not propose or the other one will not accept the proposal. Furthermore, the possible matches for $m$ are all the women of $W$ or himself (staying single). Symmetrically, the possible matches for $w$ are all the men of $M$ or herself.

A *stable matching* occurs when:

- it is individually rational, i.e., there is no $k \in M \cup W$ that finds $\mu(k)$ unacceptable; and,
- it is not blocked by any pair of agents (there is not a pair $(m, w) \in M \times W$ where each prefers each other to their current partner under $\mu$).

In other words, a matching is stable when no agent has incentives to change the current match.

After defining the problem, it is important to define the mechanism that determines a matching. A mechanism is a rule that produces a matching for any reported preferences (Kojima, 2009). For the marriage market, denote (M, W, R) by R, $\phi[R]$ the matching assigned for market R. A mechanism $\phi$ is stable and Pareto efficient if $\phi[R]$ is stable and Pareto efficient for any R.

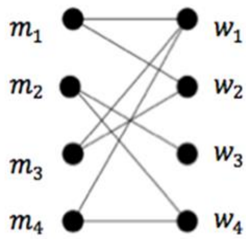
When no single player has incentives to deviate from his strategy, given that other players do not deviate, we have a Nash equilibrium (Rasmusen, 1989). In the marriage problem:

> **Theorem 1 (Gale and Sotomayor, 1985)**: When all preferences are strict, let $\mu$ be a stable matching for (M, W, R). Suppose each woman $w \in \mu\,(M)$ chooses the strategy of listing only $\mu\,(w)$ on her stated preference list of acceptable men (and each man states his true preferences). This is a Nash-equilibrium in the game induced by the men-optima Deferred Acceptance Algorithm (DAA).
>
> **Proof:** see appendix B.

A profile of strategies Q, such that each agent $k \in M \cup W$ is playing her best-response $Q_k$ to the profile of strategies of the other agents $Q_{-k}$ is a Nash equilibrium.

We shall also note that matching is related to graph theory. A graph $G$ is as a list of non-ordered pair of connected and distinct agents which constitutes the relational network between the agents (Carayol and Roux, 2009). So, given a graph G = (V, E) where V represents the vertices and E the edges, a matching is a subgraph of G where every node has degree 1 (e.g., for the marriage problem, a man can only have one wife and vice-versa) (Leighton and Rubinfeld, 2006). When all the nodes have a pair it is called a perfect matching: a matching of a graph G = (V, E) is perfect if it has $\frac{|V|}{2}$ edges.

In the case of the marriage problem, an example of a sub-graph could be:




(Adapted from Leighton and Rubinfeld, 2006)

Figure 1: A possible sub-graph of the marriage problem graph with 8 agents

where M = $\{m_1, m_2, m_3, m_4\}$ is the set of men; and W = $\{w_1, w_2, w_3, w_4\}$ is the set of women. In this case, Leighton and Rubinfeld (2006) identify $\{(m_1, w_2), (m_2, w_3), (m_3, w_1), (m_4, w_4)\}$ as a perfect matching.

Once every agent has a preference profile, every node has a preference order of the possible partners (Leighton and Rubinfeld, 2006).

Considering M $= \{m_1, m_2\}$ the set of men, W $= \{w_1, w_2\}$ the set of women and their preferences given by:

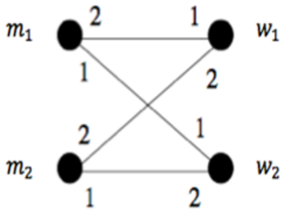
$$P_{m_1}: w_2, w_1$$

$$P_{m_2}: w_2, w_1$$

$$P_{w_1}: m_1, m_2$$

$$P_{w_2}: m_1, m_2$$

This problem can be designed as a graph:




(Adapted from Leighton and Rubinfeld, 2006)

Figure 2: A possible sub-graph of the marriage problem graph with 4 agents

After identifying the main characteristics of a matching model, the next section will focus on matching models of R&D cooperation.

### 3.3. Matching models and R&D cooperation

Recently, matching has been employed in the study of R&D networks (e.g. Li-ping, 2006; Cowan *et al*., 2009; Santamaria and Surroca, 2011). When it comes to R&D networks, the matching procedure could be decentralised or centralised. In the first case, firms negotiate directly with each other whereas, in the second case, an external entity could be created with the objective of doing the matches. Additionally, it is important to refer that the partnership can be vertical (suppliers and clients, of different markets), horizontal (rivals or competitors within the same market), with academic institutions or with foreign firms (Miotti and Sachwald, 2003; Santamaria and Surroca, 2011).

Miotti and Sachwald (2003) affirm that, according to a strategic and organisational perspective, the preference to cooperative R&D rather than developing R&D within the firm (internal R&D), equity relationships or outsourcing depends on two aspects: the characteristics of the technologies involved and the characteristics of firms' competencies. Another aspect these scholars explore is the reason why firms cooperate in R&D. They support the idea that this type of partnership can be an organisational answer to the requirements of innovation-based competition and rapid technological change. In their study the authors focus on the choice of partners. For them, cooperation with rivals is the rarest type of cooperation and is more likely to happen in high-tech sectors. For Miotti and Sachwald (2003), in this type of cooperation, firms face R&D

costs that are an obstacle to innovation and matching with each other in order to exploit economies of scale and reduce individual costs of innovation. On the other hand, vertical cooperation is relatively more frequent in low-tech sectors and involves firms that consider the lack of market information as an obstacle to innovation. This leads to cooperation with clients to alleviate these problems and increases the propensity to firms introduce new products in the market (Miotti and Sachwald, 2003). In another type of R&D cooperation, cooperation with public institutions, firms are not concentrated in R&D intensive sectors, however, they tend to be close to science resources to innovate (Miotti and Sachwald, 2003). The authors say that cooperation with rivals has the objective of combining similar resources to face high R&D costs while cooperation with universities aims at using complementary resources to work at the technological frontier. Empirical results are used in this study to demonstrate that the objectives of firms to cooperate in R&D are related to the profile of the partners they choose to team up and to the improvement of innovation. This means that the reason why firms cooperate in R&D also determine with whom they cooperate.

Santamaria and Surroca (2011) add an important contribute to the literature by examining how firms' motivations to explore new ideas or to exploit existing capabilities influence partner selection. The authors propose and test a conceptual framework that match motives to collaborate and the innovation outcomes of the partnership. Using an empirical method they conclude that when firms pursue technological objectives (exploitation and exploration), vertical collaboration (suppliers and customers) would be preferable. On the other hand, horizontal collaborations are determined, only, by exploration objectives. Forming alliances with vertical, horizontal or institutional partners have different motivations. In case of vertical collaboration, the motivation is to exploit existing competences (increase the probability of obtain product and process innovations); in case of horizontal partnership, the objective is to carry out pre-competitive research; finally, the main motivation for institutional collaboration is the exploration of new ideas (Santamaria and Surroca, 2011). So when firms try to do the perfect match for R&D purposes it is important to understand what is the motivation of such partnership, and, in that way, reach better results.

Li-ping (2006) studies the process of transferring knowledge between universities and industries. The author establishes a five-stage knowledge transfer process model that is divided in different stages: the searching stage, matching stage, learning stage, adaptation stage and integration stage. According to Szulanski (1996), Tsai (2002), and Darr and Kurtzberg (2000) (*cfr*. Li-ping, 2006), the characteristics of the required knowledge, the organisational context, the perceived reliability of the partner, the competitive relationship between the partners, the similarity between the partners and the strength of socialites are critical to do the perfect match. Partnerships between universities and firms are a particular kind of collaboration because they are heterogeneous organisations leading to a highest cooperation propensity than competitive propensity. According to Li-ping (2006), negotiation is very important in this stage and is through negotiation that the results of the research provided by the universities match the objectives of the firms.

When firms create a strategic alliance, it is essential that the participating firms resemble and complement each other, in order to achieve a profitable alliance (Cowan *et al*., 2009). To build a network, firms have to play a simultaneous link formation game. According to the study of Cowan *et al*. (2009) a strategy for firm $i$ is a list of (n-1) decisions $(s_{i,1}, \dots, s_{i,i-1}, s_{i,i+1}, \dots, s_{i,n})$, with $s_{i,j} \in \{0,1\}$. When $i$ proposes a partnership to $j$, $s_{i,j}$ will be equal to 1; whereas, when no partnership is proposed $s_{i,j}$ will be equal to 0. If $i$ proposes to $j$, an alliance is created if and only if the second firm also propose to $i$ - this means that link formation is bilateral (Cowan *et al*., 2009). According to the authors, if and only if $s_{i,j}s_{j,i} = 1$ a network *g(**s**)=g* induced by the strategy profile ***s***, $ij \in g$ is formed. All links of a stable network should yield non-negative value to both partners; if any link yields a negative value to at least one potential partner it cannot be part of the network (Cowan *et al*., 2009).

According to Jackson and Wolinsky (1996) a network $g$ is pairwise stable if and only if:

$$min\{\pi_i^g - \pi_i^{g-ij}; \pi_j^g - \pi_j^{g-ij}\} \geq 0, \forall ij \in g$$

and

$$min\{\pi_i^{g+ij} - \pi_i^g; \pi_j^{g+ij} - \pi_j^g\} \geq 0, \forall ij \in g$$

where $\pi_i^g$ represents the payoff to firm $i$ in network g.

Cowan *et al*. (2009) affirm that for a partnership to be successful, the participating firms must have the right technological fit. This means that firms must share competencies (δ) and complement their competencies (γ) in technologies. Cowan *et al*. (2009) develop a model where firms behave in a fast moving industry in which innovation is the motivating goal. For these authors, networks often share two properties: they are small worlds (short distance between pairs of agents and strong local clustering) and they have skewed link distributions. The authors consider that knowledge is modelled as a list of discrete elements where the set of all possible facts is {1, . . . , w}. When networks only have one link, it demands that each partner possesses exactly δ + γ ideas. If a partner has more or less knowledge then it cannot meet the technological fit constraint. A networks of size $s$ can be created if and only if δ + sγ ≤ w. These scholars use a heuristic way to resolve the problem. For that they start from the fact that a match between $i$ and $j$ requires that $i$ knows δ of j's δ + γ pieces of knowledge. Firms with different amounts of knowledge will never find their match.

Like Cowan *et al*. (2009), Milgram (1967), Newman (2001) and Kogut and Walker (2001) (*cfr.* Carayol and Roux, 2009) also characterised real social networks as being very short: agents who participate in a network are, on average, very close to one another; and they are highly clustered, i.e., there is a high probability that an agent's neighbours are also neighbours to one another. Among those structural properties, some authors like Gastner and Newman (2006) (*cfr.* Carayol and Roux, 2009) also studied the spatial distribution of networks and demonstrated that some of them tend to be correlated with geography because they show an inverse relationship between geographical distance and social ties.

Carayol and Roux (2009) introduced a model of network formation of agents who balance the benefits of forming links against their costs, which linearly increase with geographic distance. On their work, the authors started to define a network formation as a game where pairs of agents meet and decide to form, maintain or break links. The formation of a link is bilateral (both agents have to agree in the partnership) but the sever of a link is unilateral. According to the same authors, and due to myopia, agents make their decisions considering the immediate impacts on their current payoffs. Let $\pi_i: \{g | g \subseteq g^N\} \to \mathbb{R}$ represent the payoffs received by $i$ from is positions on the network g considering the complete graph $g^N = \{ij | i, j \in N\}$ as the set of all subsets of

N of size 2 where each agent is connected with all others and $g \subseteq g^N$ an arbitrary collection of links on N.

To analyse the network efficiency, Carayol and Roux (2009) use the notion introduced by Jackson and Wolinsky (1996), which is the total value of a graph. The total value of a graph g is given by $\pi(g) = \sum_{i \in N} \pi_i(g)$ and a network g is efficient if it maximises this sum in the set of all possible graphs $\{g | g \subseteq g^N\}$: this means that $\pi(g) \geq \pi(g')$ for all $g' \subseteq g^N$.

To demonstrate that the strategic approach to link formation can generate networks that share some of the main structural properties of most real social networks, Carayol and Roux (2009) introduce a strategic model of network formation built on a simple variation of the connections model of Jackson and Wolinsky (1996). In that model, agents benefit from knowledge that flows through bilateral relationships. However, as bigger is the relational distance between the agents, the lower is the positive externality; and as bigger is the geographic distance, the higher are the costs. Carayol and Roux (2009) formalised the net profit received by any agent $i$ as:

$$\pi_i(g) = \sum_{j \in N \setminus i} \delta^{d(i,j)} \omega_{ij} - \sum_{j: ij \in g} c_{ij}$$

where $d(i, j)$ is the geodesic distance between $i$ and $j$; $c_{ij}$ the cost borne by $i$ for a direct connection with $j$; $\omega_{ij}$ is the "intrinsic value" of individual $j'$s knowledge to individual $i$ (for simplicity let $\omega_{ij}$ be one: $\forall i \neq j: \omega_{ij} = \omega = 1$; and $\delta \in ]0,1[$ is the decay parameter representing the share of knowledge effectively transmitted through each edge. The costs of maintaining a direct connection between $i$ and $j$ is equal to the geographic distance separating them:

$$c_{ij} = s_{ij} = l(i, j)\lceil n/2 \rceil^{-1}$$

Focusing on the network formation, let's consider $g_t \in G$ as the state of the social network at period $t$ (with $t = 1,2, ...$). Two agents $i$ and $j \in N$ are randomly selected at each time period and are given a jointly decision to make: maintain or unilaterally break the link between them, if they are directly connected; or bilaterally form a link or unilaterally decide against it if they are not connected. Formally:

(i) if $ij \in g_t$, the link is maintained if $\pi_i(g_t) \geq \pi_i(g_t - ij)$ and $\pi_j(g_t) \geq \pi_j(g_t - ij)$. Otherwise, the link is deleted.

(ii) if $ij \notin g_t$, a new link is created if $\pi_i(g_t + ij) \geq \pi_i(g_t)$ and $\pi_j(g_t + ij) \geq \pi_j(g_t)$, with a strict inequality for one of them.

With this model Carayol and Roux (2009) were able to conclude that for intermediate levels of knowledge transferability, clustering occurs in geographical space and a few agents sustain distant connections. According to them, that type of networks has the small world property.

Campos *et al*. (2013) used an agent-based model in a R&D network context. In their work, the authors compared three collaboration strategies: peer-to-peer complementariness; concentration process; and virtual cooperation networks. Campos *et al*. (2013) conclude that profit is associated with higher knowledge stock and with smaller network diameter. When comparing all strategies, the authors conclude that concentration strategies are more profitable and more efficient in transmitting knowledge through the network.

After analysing literature on matching models and focusing on the simplest case of two-sided matching, the next chapter will focus on the development of a matching model of R&D cooperation, which is the main subject of this dissertation.

# Chapter 4 - A matching model of R&D cooperation

In this chapter it will be developed a matching model of R&D cooperation in which firms make their alliance decisions based on the technological fit of potential partners. By cooperating with each other in R&D activities, firms can achieve some advantages like cost-reducing technology and, therefore, gain competitive advantages.

For that purpose, inspiration is found in the models developed by Goyal and Moraga-González (2001), Cowan *et al*. (2009) and Campos *et al*. (2013).

### 4.1. The Model

Let's represent the set of firms as $N = \{1, \ldots, n\}, n \geq 2$. A binary variable $g_{ij} \in \{0, 1\}$ represents the pair-wise relationship between any pair of firms $i, j \in N$. When $g_{ij} = 1$ this means that the two firms are linked, while $g_{ij} = 0$ refers to the case where firms are not linked.

A network $g$ is a collection of links, i.e., $g = \{g_{ij}\}_{i,j \in N}$. When a link is added between firms $i$ and $j$ to a network $g$ is represented by $g + g_{ij}$. On the other hand, $g - g_{ij}$ represents the sever of a link between firms $i$ and $j$ from network $g$.

The firms with whom $i$ is directly connected are its neighbours and that is denoted by $N_i^g = \{j \neq i : ij \in g\}$. The size of the neighborhood of $i$ (or its degree) is the number of links held by firm $i$ and is denoted by $n_i^g = \# N_i^g$.

The total number of links in the network $g$ is $E^g = \sum_{i \in N} \frac{n_i^g}{2}$, and the density of $g$ is equal to $\frac{2E^g}{n\,(n-1)}$. When a firm $i$ has no partner is called a *singleton* and, in this case, the number of links held by firm $i$ is represented by $n_i^g = 0$ (Cowan *et al*., 2009).

A market may have more than one network so we denote G as a finite set of all networks that exist in the market, $G = \{g \subseteq g^N\}$.

To build a network, firms have to play a simultaneous link formation game. Firm $i$, for example, has to decide whether to create or not a link with firm $j$ or, if the link is already created, firm $i$ has to decide to maintain or sever the link. A strategy for firm $i$ is a list of $n-1$ decisions $s_i = (s_{i,1}, \dots, s_{i,i-1}, s_{i,i+1}, \dots, s_{i,n})$ with $s_{i,j} \in \{0,1\}$. When $s_{i,j} = 1$ , $i$ proposed a partnership to $j$ but when $s_{i,j} = 0$ no partnership is proposed. As it was mentioned in section 3.3., a matching decision must be bilateral so an alliance is only formed if both firms want that to happen and propose to each other. Mathematically: $g(s) = g$ induced by the strategy profile $s = (s_1, s_2, \dots, s_n)$ if and only if $s_{i,j}, s_{j,i} = 1, i, j \in g$ .

At any period $t$ (with $t = 1, 2, \dots$), firms can decide to form, maintain or delete links in the following conditions, based on the value of $\pi_i$ that represents firm $i$'s profit:

(i) when $g_{ij} = 1$, the link is maintained if $\pi_i^g \geq \pi_i(g - g_{ij})$ and $\pi_j^g \geq \pi_j(g - g_{ij})$. Otherwise the link is deleted;

(ii) when $g_{ij} = 0$, a new link is created if $\pi_i\big(g + g_{ij}\big) \geq \pi_i^g$ and $\pi_j(g + g_{ij}) \geq \pi_j^g$.

The evolution of the network at any time $t$ depends only on the present state of the network given by the graph structure of $g$.

As defined in section 3.3., a network g is pairwise stable if and only if (Jackson and Wolinsky, 1996):

$$min\left\{\pi_i^g - \pi_i^{g-g_{ij}}; \pi_j^g - \pi_j^{g-g_{ij}}\right\} \geq 0, \forall ij \in g$$

and

$$min\left\{\pi_i^{g+g_{ij}} - \pi_i^g; \pi_j^{g+g_{ij}} - \pi_j^g\right\} \geq 0, \forall ij \in g$$

where $\pi_i^g$ represents the payoff of firm $i$ in network g.

In our model we will define the knowledge stock of firm $i$ at time $t$ ($k_i^t$) as following:

$$k_i^t = k_i^{t-1} \times (1-\delta) + x_i + \sum_{k \neq i \in N_i^g} \beta(d_{ik}) \times x_k + \sum_{l \notin N_i^g} (\beta(d_{il}))^6 \times x_l$$

where $x_i$ represents the R&D effort made by firm $i$ and $\delta$ represents the knowledge stock deterioration rate over time. It is important to note that, a firm $i$ only develops R&D activity in period $t$ if, in $t-1$, $\pi_i > 0$.

The knowledge stock of firm $i$ is then due the knowledge of firm $i$ in previous period of time (taking into account the deterioration rate) plus the flow of knowledge obtained in time $t$, which is due to:

(i) firm $i$ own R&D research ($x_i$);

(ii) the R&D efforts developed by other firms with whom firm $i$ has a collaborative link ($x_k$ with $k \neq i \epsilon N_i^g$); and

(iii) the research of other firms with whom firm $i$ has no link ($x_l$ with $l \notin N_i^g$) but that partially spillovers to firm $i$.

In this research, we will assume that the level of external R&D absorbed by firm $i$ also depends of the technological distance between firms ($d_{ij}$) and is introduced through the spillover function $\beta = f(d_{ij})$. For the same technological distance, a firm that belongs to a network will absorb higher levels of R&D produced by his partners, $\beta(d_{ik})$, than the R&D produced by the outside firms, $(\beta(d_{il}))$[6]. This happens because partner firms can share information more easily. In order to evaluate the technological fit of potential partners, we will consider the technological distance between firms $i$ and $j$ at time $t$ ($d_{ij}$), which, according to Campos *et al*. (2013), represents the difference of the technological know-how between two organizations.

This variable is measured by the difference of firms' knowledge stock in each period $t$ ($k_i^t, k_j^t$), while $Max\ (k^t)$ is the maximum knowledge stock owned by any firm in the industry (Campos *et al*., 2013):

$$d_{ij} = \frac{\left|k_i^t - k_j^t\right|}{Max\ (k^t)}$$

We expect that two firms will cooperate if the difference between their knowledge stocks is neither to large or too small. According to D'Ágata and Santangelo (2003):

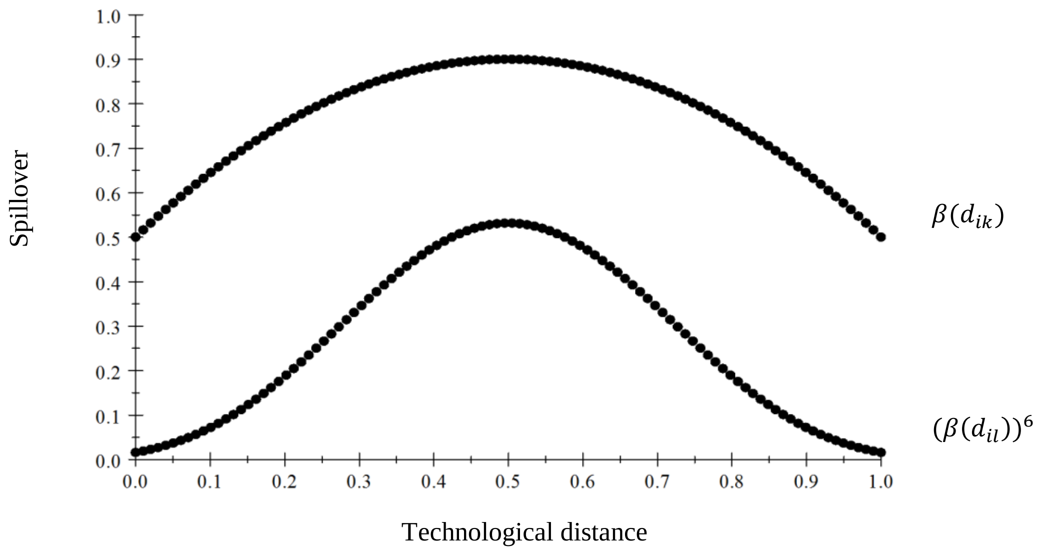
(i) a small cognitive distance allows greater comprehensibility, but yields redundant, novel knowledge;

(ii) a large cognitive distance allows limited comprehensibility, although yielding non-redundant, novel knowledge;

(iii) a certain degree of cognitive distance is needed since it ensures that firms can connect their cognitive frameworks and being innovative as well as they can easily communicate between each other's.

This reasoning is captured by introducing a quadratic relation between the R&D spillover $\beta$ and the technological distance $(d_{ij})$ between firms:

$$\beta(d_{ij}) = a_0 + a_1 d_{ij} + a_2 {d_{ij}}^2 \text{, with } a_2 < 0$$




Legend: $d_{ik}$ = distance between cooperating firms *i* and *k*; $d_{il}$ = distance between non-cooperating firms *i* and *l*.

Figure 3: Technological distance and spillover

If two firms have a collaboration link, the spillover is higher than in the case where no link exists, given the same technological distance. In addition, firms with non-extreme values of technological distance absorb higher levels of R&D produced by other firms.

In our model, the R&D effort influences the cost of production. Considering the existence of R&D spillovers, then the R&D effort made by one firm decreases its own production cost and may also decrease other firm's production cost.

The production cost of firm $i$, $c_i^g$, is then defined as follows:

$$c_i^g = \bar{c} - x_i - \sum_{k \neq i \in N_i^g} \beta(d_{ik}) \times x_k - \sum_{l \notin N_i^g} (\beta(d_{il}))^6 \times x_l$$

where $\bar{c}$ is the stand-alone constant marginal cost.

However, developing R&D activities have a cost so, given a level $x_i \epsilon [0, \bar{c}]$ of R&D effort, the associated cost is:

$$Z(x_i) = \gamma x_i^2, \qquad \gamma > 0$$

where the parameter $\gamma$ measures the curvature of the R&D cost function (Goyal and Moraga-González, 2001). Therefore, the R&D cost is an increasing function and exhibits decreasing returns.

Finally, and assuming that firms choose quantities $\{q_i^g\}_{i \epsilon N}$ in the product market and that the demand is linear and given by $Q = a - p, a > \bar{c}$, then the profit of firm $i$ in a collaborative network $g$ is:

$$\pi_i^g = \left[a - q_i - \sum_{j \neq i} q_j - c_i^g\right] q_i - \gamma x_i^2$$

### 4.2. An analytical resolution

In this section we will present an analytical resolution of the model for the duopoly case. We propose a two-stage game where firms decide about R&D and production. The timing is the following:

1$^{st}$) Firms simultaneously choose their R&D effort level ($x_i$), independently or under cooperation;

2$^{nd}$) Firms simultaneously choose the level of output ($q_i$), through Cournot competition.

The game is solved by backward induction to ensure sub-game perfectness and we consider two alternative scenarios: R&D competition, where firms choose their R&D output independently, and R&D cooperation, where firms cooperate and coordinate R&D output in order to maximise joint profits.

#### 4.2.1. Competition in R&D

In this case, firms do not belong to the network, in spite of benefiting from a R&D spillover. For simplicity, we assume $\gamma = 0.5$.

Firm *i*'s profit function is given by:

$$\pi_i(q_1, q_2, x_i) = (a - q_1 - q_2 - c_i)q_i - 0.5x_i^2$$

and firm *i*'s cost function is given by:

$$c_i^g(q_1, q_2, x_i, x_j) = \bar{c} - x_i - \beta(d_{1,2}) \times x_j$$

where $\beta(d_{1,2})$ is the spillover between both firms that depends on their technological distance.

Consequently we have:

$$\pi_i(q_1, q_2, x_i, x_j) = (a - q_1 - q_2 - \bar{c} + x_i + \beta \times x_j)q_i - 0.5x_i^2$$

From the Cournot game it is straightforward to determine firm $i$'s best response:

$$\frac{\partial \pi_1(q_1, q_2, x_1, x_2)}{\partial q_1} = 0 \Leftrightarrow q_1^* = \frac{a - q_2 - \bar{c} + x_1 + \beta \times x_2}{2}$$
$$\frac{\partial \pi_2(q_1, q_2, x_1, x_2)}{\partial q_2} = 0 \Leftrightarrow q_2^* = \frac{a - q_1 - \bar{c} + x_2 + \beta \times x_1}{2}$$

and therefore output equilibrium

$$\left( \begin{array}{c} q_1^* = \dfrac{a - \bar{c} + 2x_1 - x_2 - \beta x_1 + 2\beta x_2}{3}, \\ q_2^* = \dfrac{a - \bar{c} - x_1 + 2x_2 + 2\beta x_1 - \beta x_2}{3} \end{array} \right)$$

Firms' R&D effort equilibrium is given by maximising the following profit functions:

$$\pi_1(q_1^*, q_2^*, x_1, x_2) = \frac{(a - \bar{c} + 2x_1 - x_2 - \beta x_1 + 2\beta x_2)^2}{9} - 0.5x_1^2$$
$$\pi_2(q_1^*, q_2^*, x_1, x_2) = \frac{(a - \bar{c} + 2x_2 - x_1 - \beta x_2 + 2\beta x_1)^2}{9} - 0.5x_2^2$$

R&D output equilibrium is then given by:

$$\left(x_1^* = x_2^* = \frac{(a-\bar{c})(2-\beta)}{\beta^2-\beta+2.5}\right)$$

### 4.2.2. R&D cooperation

In this case, firms belong to a R&D network. For simplicity, will assume $\gamma = 0.5$.

In the R&D stage, cooperation implies that each firm within the R&D cartel will choose its R&D output in order to maximise joint profits:

$$\pi_1(q_1^*, q_2^*, x_1, x_2) + \pi_2(q_1^*, q_2^*, x_1, x_2)$$

The R&D output equilibrium in then given by:

$$\left(x_1^{**} = x_2^{**} = \frac{(a-\bar{c})(\beta+1)}{-\beta^2-2\beta+3.5}\right)$$

which is in accordance with d'Aspremont and Jacquemin (1988).

**Example**: Suppose a = 3,000, $\bar{c}$= 300.

In the cooperative scenario, and assuming $\beta$ = 0.89, then $x_1^{**} = x_2^{**} = 5{,}500$ and $q_1^{**} = q_2^{**} = 4{,}365$. In the competitive case, and assuming $\beta = 0.5 \cong 0.89$[6], we have $x_1^* = x_2^* = 1{,}800$ and $\mathrm{q}_1^* = \mathrm{q}_2^* = 1{,}800$.

The main conclusion of this analytical resolution is that, in a cooperative scenario, each firm has higher R&D and output levels than in the competitive case. The intuition is rather simple: when R&D runs cooperatively, R&D equilibrium output increases with the degree of information sharing between firms, leading to a reduction in the productions costs and turning the increase in the output produced more appealing.

This result is also evidenced in works such as d'Aspremont and Jacquemin (1988) or Kamien et al. (1992). In fact, d'Aspremont and Jacquemin (1988) concluded that, for large spillovers, firms that cooperate in R&D can achieve higher levels of R&D and can increase the quantity produced. Similarly, Kamien et al. (1992) concluded that a RJV that cooperates in its R&D expenditure decision yields the highest consumer plus producer surplus under Cournot competition.

## 4.3. Simulation results through an agent-based model

In the previous section we consider a duopoly scenario with homogeneous firms. In this section, we will consider heterogeneous agents and a market with 10 firms, bringing complexity to the model and making it impossible to be solved analytically. Thus, in order to accomplish the main objective of this dissertation, it will be develop an agent-based model that allows us to explore the evolution of firms' alliances for R&D purposes by means of simulation. Therefore, agent-based models are particularly useful when dealing with heterogeneous agents, *t* periods and *n* firms.

According to Huhns and Singh (1998) (*cfr.* Gilbert and Troitzsch, 1999) the term agent is usually used to describe self-contained programs that can control their own actions based on their perception of their operating environment. According to Ferber (1999), an agent can be a physical or a virtual entity that can act, perceive its environment and communicate with others. Additionally, an agent is autonomous and has skills to achieve its goals and tendencies.

For Damaceanu (2010) (*cfr.* Damaceanu, 2013), an agent-based computational model or just Agent-Based Model (ABM) is a branch of Applied Computational Mathematics which is the field that studies the construction of mathematical models and numerical solution techniques by using computers. ABM are useful to analyse and solve social and engineering problems. In his recent study, Damaceanu (2013) defines an agent-based model as a model of a real economic system that takes input data and creates output data by running computer experiments.

In what concerns the formation of networks, authors like Jackson and Wolinsky (1996), Carayol and Roux (2009), Cowan *et al*. (2009) and Campos *et al*., (2013) also used simulated environments to study economic phenomena.

### 4.3.1. Simulation results

This dissertation introduces an agent-based model (see pseudo code in appendix C) in order to study the relation between network formation and the technological fit between potential partners. The software used to produce simulation results was R.

In this study, firms are characterised by a technological profile and may create networks for R&D purposes. To accomplish our objectives and avoid excessive complexity, we will analyse the formation of networks over 5 periods of time ($t = 1, \ldots, 5$), considering a sample of 10 firms and assuming that, in the first network, the connecting ratio to create a link is 40% (meaning that, there is a 40% probability of a firm to create a link at $t = 1$). On table 1 we list the key parameters assumed in our simulation:

Table 1: Key parameters

| **Parameter** | **Value** |
|---|---|
| $a$ | 3000 |
| $\bar{c}$ | [100; 300] |
| $a_0$ | 0.5 |
| $a_1$ | 1.6 |
| $a_2$ | -1.6 |
| $\gamma$ | 0.5 |
| $\delta$ | 0.02 |

The initial values of each firm's knowledge stock ($k_i^0$) and output ($q_i^0$) are obtained as a random value from the Uniform distribution within the range [1; 500] and [1; 50], respectively, providing stochasticity to the model.

Table 2: Initial values

| **Variables** | **Range of values** |
|---|---|
| $k$ | U[1;500] |
| $q$ | U[1;50] |

After running the model in R, we obtain the adjacency matrix, a symmetric matrix (10x10) for each iteration that gives us information about the links established between firms. The value 1 means that firms *i* and *j* have a link and 0 means that there is no link between them. As explained in the previous sections, the decision of maintaining, breaking or creating a link is bilateral so if firm $i$ has a link with firm $j$ then firm $j$ has also a link with firm $i$ and so, the result is a symmetric matrix. For the first iteration

($t = 1$) we have the following results (which can be distinct in different runs of the simulations due to the stochasticity of the model):

Table 3: Adjacency matrix (10x10) when t=1

| **Firm** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| **2** | 0 | | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| **3** | 0 | 0 | | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| **4** | 0 | 1 | 1 | | 0 | 0 | 0 | 0 | 0 | 0 |
| **5** | 0 | 1 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 |
| **6** | 0 | 1 | 0 | 0 | 0 | | 1 | 0 | 0 | 0 |
| **7** | 0 | 0 | 0 | 0 | 0 | 1 | | 0 | 0 | 0 |
| **8** | 1 | 0 | 1 | 0 | 0 | 0 | 0 | | 0 | 0 |
| **9** | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | | 0 |
| **10** | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | |

In the first network produced by this model we have the following graph:

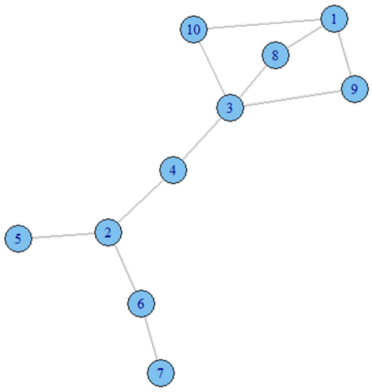


Figure 4: Network when t=1

By analysing Figure 4, we can see that all firms are inside the network; firm 5 and 7 have only one link; firms 4, 6, 8, 9 and 10 have 2 connections each; firms 1 and 2 created 3 links; and, firm 3, with 4 connections formed, is the one with more links inside the network .

However, this process is dynamic so, if we analyse the last iteration ($t = 5$) we may have a different network:

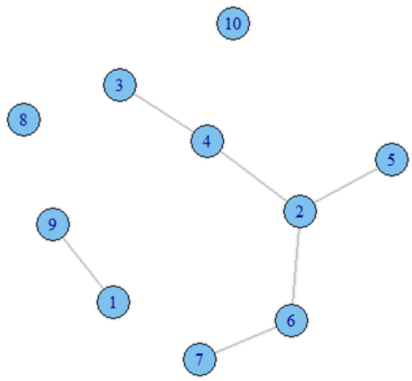



Figure 5: Network when t=5

At $t = 5$, there are two networks formed and two firms are outside the networks (firms 8 and 10).

Analysing the evolution of the networks over the 5 iterations we have the following:

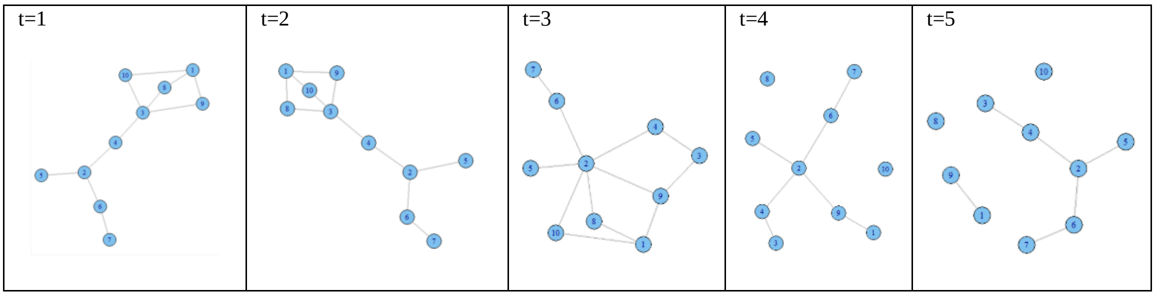



Figure 6: Network evolution

The networks formed in the two first iterations are identical. In the third iteration, firms 2 and 9 create some additional links and firm 3 severed 2 links when comparing to the network formed in $t = 1$ and $t = 2$. At $t = 4$, the majority of the firms broke some connections and at $t = 5$, there are two networks as mentioned above.

In order to better analyse the networks, we calculate three statistical measures: degree centralisation, betweenness centralisation and closeness centralisation.

The **degree centralisation** is a network most basic structural property, as it measures the number of each node's adjacent edges (Csardi, 2014). So, the nodes with higher degree are more central. According to Du (2015), firms with more connection tend to have more influence or importance within the network. For example, in Figure 7 we represent each firm's connections for $t = 1$ and thus conclude that firm 3 is the more central firm with a degree centralisation of 4:

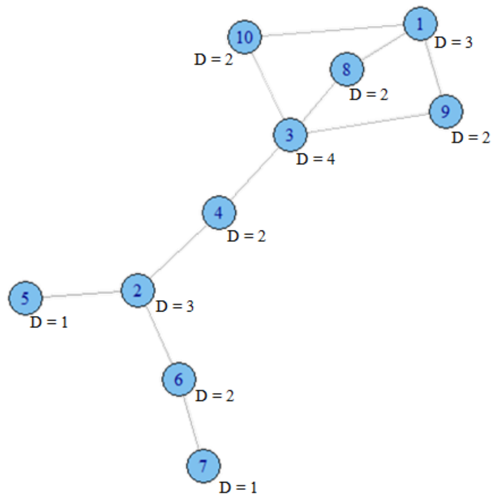



Figure 7: Degree (t=1)

In table 4 we can find the degree centralisation evolution over the 5 iterations. We can observe that firms 2 and 3 are the most central nodes, while firms 5 and 7 is the less central firm.

Table 4: Degree centralisation

| t \ Firm | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 3 | 3 | 4 | 2 | 1 | 2 | 1 | 2 | 2 | 2 |
| **2** | 3 | 3 | 4 | 2 | 1 | 2 | 1 | 2 | 2 | 2 |
| **3** | 3 | 6 | 2 | 2 | 1 | 2 | 1 | 2 | 3 | 2 |
| **4** | 1 | 4 | 1 | 2 | 1 | 2 | 1 | 0 | 2 | 0 |
| **5** | 1 | 3 | 1 | 2 | 1 | 2 | 1 | 0 | 1 | 0 |
| **Average** | 2.2 | 3.8 | 2.4 | 2 | 1 | 2 | 1 | 1.2 | 2 | 1.2 |

According to Csardi (2014), the vertex and edge **betweenness centralisation** are (roughly) defined by the number of geodesics (shortest paths) going through a vertex or an edge. Therefore, betweenness centralisation quantifies the number of times a node acts as a bridge along the shortest path between two other nodes (Du, 2015). The betweenness of a vertex v in a graph G can be represented as follows:

$$Betwenness\ (v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where $\sigma_{st}$ is total number of shortest paths from node s to node t and $\sigma_{st}(v)$ is the number of those paths that pass through v.

In table 5 we can find the betweenness centralisation evolution over the 5 iterations:

Table 5: Betweenness centralisation

| t \ Firm | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 2 | 20 | 22 | 20 | 0 | 8 | 0 | 2 | 2 | 2 |
| **2** | 2 | 20 | 22 | 20 | 0 | 8 | 0 | 2 | 2 | 2 |
| **3** | 2 | 26 | 1 | 3 | 0 | 8 | 0 | 2 | 6 | 2 |
| **4** | 0 | 18 | 0 | 6 | 0 | 6 | 0 | 0 | 6 | 0 |
| **5** | 0 | 8 | 0 | 4 | 0 | 4 | 0 | 0 | 0 | 0 |

By analysing the table above we may conclude that:

- at $t = 1$ and $t = 2$, firm 2, 3 and 4 are the most centralised firms;
- at $t = 3$ and $t = 4$, firm 2 is the most centralised firm;
- at $t = 5$ firm 2 is the most centralised firm followed by firms 4 and 6.

In sum, firm 2 is the most centralised firm meaning that, most of the times, the connections between the other firms are made through firm 2.

**Closeness centralisation** measures how many steps are required to access every other vertex from a given vertex and is defined by the inverse of the average length of the shortest paths to/from all the other vertices in the graph (Csardi, 2014). The more central a node is the lower its total distance to all other nodes (Du, 2015).

$$Closeness = \frac{1}{\sum_{i \neq v} d_v i}$$

Table 6: Closeness centralisation

| t \ Firm | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0.036 | 0.050 | 0.056 | 0.056 | 0.036 | 0.038 | 0.029 | 0.042 | 0.042 | 0.042 |

| Firm / t | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **2** | 0.036 | 0.050 | 0.056 | 0.056 | 0.036 | 0.038 | 0.029 | 0.042 | 0.042 | 0.042 |
| **3** | 0.050 | 0.083 | 0.045 | 0.056 | 0.050 | 0.056 | 0.038 | 0.056 | 0.063 | 0.056 |
| **4** | 0.025 | 0.033 | 0.025 | 0.029 | 0.028 | 0.029 | 0.025 | 0.011 | 0.029 | 0.011 |
| **5** | 0.012 | 0.021 | 0.019 | 0.020 | 0.020 | 0.020 | 0.019 | 0.011 | 0.012 | 0.011 |

In the first two iterations, firms 3 and 4 are the closeness central nodes. At $t = 4$ and $t = 5$, firm 2 is the one with the lowest distance to all other nodes.

### 4.3.2. Statistical analysis

In the section before, we made some assumptions for some parameters (section 4.3.1., Table 1). Since different values could lead to quantitatively different results, we ran the simulation 7 times for the baseline parameters, and also considering different values for the key parameters:

Table 7: Key parameters - values for simulation

| Number of simulations | Number of firms | Number of iterations | a | γ | Connecting ratio |
|---|---|---|---|---|---|
| 7 | 10 | 5 | 3000 | 0.5 | 40% |
| 7 | 5 | 5 | 3000 | 0.5 | 40% |
| 7 | 7 | 5 | 3000 | 0.5 | 40% |
| 7 | 12 | 5 | 3000 | 0.5 | 40% |
| 7 | 15 | 5 | 3000 | 0.5 | 40% |
| 7 | 10 | 1 | 3000 | 0.5 | 40% |
| 7 | 10 | 3 | 3000 | 0.5 | 40% |
| 7 | 10 | 8 | 3000 | 0.5 | 40% |
| 7 | 10 | 10 | 3000 | 0.5 | 40% |
| 7 | 10 | 5 | 2500 | 0.5 | 40% |
| 7 | 10 | 5 | 2750 | 0.5 | 40% |
| 7 | 10 | 5 | 3250 | 0.5 | 40% |
| 7 | 10 | 5 | 3500 | 0.5 | 40% |
| 7 | 10 | 5 | 3000 | 0.3 | 40% |
| 7 | 10 | 5 | 3000 | 0.4 | 40% |
| 7 | 10 | 5 | 3000 | 0.6 | 40% |
| 7 | 10 | 5 | 3000 | 0.7 | 40% |

| Number of simulations | Number of firms | Number of iterations | a | γ | Connecting ratio |
|---|---|---|---|---|---|
| 7 | 10 | 5 | 3000 | 0.5 | 30% |
| 7 | 10 | 5 | 3000 | 0.5 | 35% |
| 7 | 10 | 5 | 3000 | 0.5 | 45% |
| 7 | 10 | 5 | 3000 | 0.5 | 50% |

The results for the last iteration of the 147 simulations are resumed in annex 3.

Computing the Spearman rank correlation[1] among variables using the SPSS program, we obtain the following results:

Table 8: Spearman rank correlation

| | | Number of links | R&D investment | Profit | Output | Tech distance | Degree | Betweenness | Closeness |
|---|---|---|---|---|---|---|---|---|---|
| Number of links | ρ | 1 | 0.247** | -0.087 | 0.277** | 0.537** | 0.978** | 0.840** | 0.692** |
| | P-value | - | 0.003 | 0.297 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| R&D investment | ρ | 0.247** | 1 | 0.578** | 0.784** | 0.343** | 0.274** | 0.288** | 0.355** |
| | P-value | 0.003 | - | 0.000 | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 |
| Profit | ρ | -0.087 | 0.578** | 1 | 0.321** | 0.183* | -0.028 | -0.021 | 0.323** |
| | P-value | 0.297 | 0.000 | - | 0.000 | 0.027 | 0.734 | 0.804 | 0.000 |
| Output | ρ | 0.277** | 0.784** | 0.321** | 1 | 0.171* | 0.299** | 0.268** | 0.287** |
| | P-value | 0.001 | 0.000 | 0.000 | - | 0.038 | 0.000 | 0.001 | 0.000 |
| Tech distance | ρ | 0.537** | 0.343** | 0.183* | 0.171* | 1 | 0.592** | 0.514** | 0.779** |
| | P-value | 0.000 | 0.000 | 0.027 | 0.038 | - | 0.000 | 0.000 | 0.000 |
| Degree | ρ | 0.978** | 0.274** | -0.028 | 0.299** | 0.592** | 1 | 0.820** | 0.768** |
| | P-value | 0.000 | 0.001 | 0.734 | 0.000 | 0.000 | - | 0.000 | 0.000 |
| Betweenness | ρ | 0.840** | 0.288** | -0.021 | 0.268** | 0.514** | 0.820** | 1 | 0.658** |
| | P-value | 0.000 | 0.000 | 0.804 | 0.001 | 0.000 | 0.000 | - | 0.000 |
| Closeness | ρ | 0.692** | 0.355** | 0.323** | 0.287** | 0.779** | 0.768** | 0.658** | 1 |
| | P-value | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | - |

* ρ significant at 0.05 (bilateral); * * ρ significant at 0.01 (bilateral).

[1] The Spearman rank correlation coefficient ρ (rho) is often used as a test statistic to test for independence between two random variables (Conover, 1980).

Analysing the table above, we might conclude that there is a strong positive correlation between the *R&D investment* and *Profit* (0.578). Intuition for this result is rather simple: the highest the profit of a firm, the highest investment the firm might do in R&D activity and vice-versa. This result is in accordance with Campos *et al*. (2013) who concluded that profit is associated with higher stock of knowledge and with smaller network diameter.

The *R&D investment* is also positively correlated with the *Output* (0.784). In fact, when investing in R&D activities, the cost of production reduces making it more appealing to produce more output. We also observe that there is a positive correlation between *R&D investment* and the *centrality* (*Degree, Betweenness* or *Closeness)*. This result is rather interesting, revealing that investment in R&D activities influences the position of a firm in the network. This could be explained through the positive correlation between the *R&D investment* and the *number of links*. Additionally, firms' *technological distance* is positively correlated with R&D investment.

In addition, there is a positive correlation between *technological distance* and the *number of links* (0.537): *ceteris paribus*, if the number of links increases, then the technological distance also increases. As a result, *centrality* is also positively correlated with the *technological distance*. Cowan *et al.* (2009) and Carayol and Roux (2009) also refer the relevance of the technological fit of potential partners on the network clustering, which is related with centrality.

We also test our results using the Kruskal-Wallis test in order to analyse the impact of the independent variables (*number of firms, number of iterations*, $a, \gamma$, *connecting ratio*) in the dependent variables (*number of links, R&D investments, profit, output, technological distance, centrality measures*), once more with the support of the SPSS program.

According to Conover (1980), in the Kruskal-Wallis test, the experimental situation is one where $k$ random samples have been obtained, one from each of $k$ possibly diferent populations, and we want to test the null hypothesis that all the populations are identical against the alternative that some the populations tend to furnish greater observed values that other populations.

Table 9: Kruskal-Wallis test

| Null hypothesis | | P-Value | Decision* |
|---|---|---|---|
| $H1_0$ | Number of links distribution is the same between the categories of Number firms | 0.000 | Reject $H1_0$ |
| $H2_0$ | R&D investment distribution is the same between the categories of Number firms | 0.000 | Reject $H2_0$ |
| $H3_0$ | Profit distribution is the same between the categories of Number firms | 0.000 | Reject $H3_0$ |
| $H4_0$ | Output distribution is the same between the categories of Number firms | 0.000 | Reject $H4_0$ |
| $H5_0$ | Tech distance distribution is the same between the categories of Number firms | 0.000 | Reject $H5_0$ |
| $H6_0$ | Degree distribution is the same between the categories of Number firms | 0.000 | Reject $H6_0$ |
| $H7_0$ | Betweenness links distribution is the same between the categories of Number firms | 0.000 | Reject $H7_0$ |
| $H8_0$ | Closeness distribution is the same between the categories of Number firms | 0.000 | Reject $H8_0$ |
| $H9_0$ | Number of links distribution is the same between the categories of Number iterations | 0.000 | Reject $H9_0$ |
| $H10_0$ | R&D investment distribution is the same between the categories of Number iterations | 0.000 | Reject $H10_0$ |
| $H11_0$ | Profit distribution is the same between the categories of Number iterations | 0.000 | Reject $H11_0$ |
| $H12_0$ | Output distribution is the same between the categories of Number iterations | 0.000 | Reject $H12_0$ |
| $H13_0$ | Tech distance distribution is the same between the categories of Number iterations | 0.000 | Reject $H13_0$ |
| $H14_0$ | Degree distribution is the same between the categories of Number iterations | 0.000 | Reject $H14_0$ |
| $H15_0$ | Betweenness links distribution is the same between the categories of Number iterations | 0.000 | Reject $H15_0$ |
| $H16_0$ | Closeness distribution is the same between the categories of Number iterations | 0.000 | Reject $H16_0$ |
| $H17_0$ | Number links distribution is the same between the categories of a | 0.000 | Reject $H17_0$ |
| $H18_0$ | R&D investment distribution is the same between the categories of a | 0.000 | Reject $H18_0$ |
| $H19_0$ | Profit distribution is the same between the categories of a | 0.000 | Reject $H19_0$ |
| $H20_0$ | Output distribution is the same between the categories of a | 0.000 | Reject $H20_0$ |
| $H21_0$ | Tech distance distribution is the same between the categories of a | 0.001 | Reject $H21_0$ |
| $H22_0$ | Degree distribution is the same between the categories of a | 0.000 | Reject $H22_0$ |
| $H23_0$ | Betweenness links distribution is the same between the categories of a | 0.000 | Reject $H23_0$ |
| $H24_0$ | Closeness distribution is the same between the categories of a | 0.000 | Reject $H24_0$ |
| $H25_0$ | Number of links distribution is the same between the categories of $\gamma$ | 0.865 | Retain $H25_0$ |
| $H26_0$ | R&D investment distribution is the same between the categories of $\gamma$ | 0.963 | Retain $H26_0$ |
| $H27_0$ | Profit distribution is the same between the categories of $\gamma$ | 0.010 | Reject $H27_0$ |
| $H28_0$ | Output distribution is the same between the categories of $\gamma$ | 0.567 | Retain $H28_0$ |
| $H29_0$ | Tech distance distribution is the same between the categories of $\gamma$ | 0.171 | Retain $H29_0$ |
| $H30_0$ | Degree distribution is the same between the categories of $\gamma$ | 0.765 | Retain $H30_0$ |
| $H31_0$ | Betweenness links distribution is the same between the categories of $\gamma$ | 0.327 | Retain $H31_0$ |
| $H32_0$ | Closeness distribution is the same between the categories of $\gamma$ | 0.135 | Retain $H32_0$ |
| $H33_0$ | Number of links distribution is the same between the categories of Connecting ratio | 0.348 | Retain $H33_0$ |

| Null hypothesis | | P-Value | Decision* |
|---|---|---|---|
| $H34_0$ | R&D investment distribution is the same between the categories of Connecting ratio | 0.902 | Retain $H34_0$ |
| $H35_0$ | Profit distribution is the same between the categories of Connecting ratio | 0.321 | Retain $H35_0$ |
| $H36_0$ | Output distribution is the same between the categories of Connecting ratio | 0.404 | Retain $H36_0$ |
| $H37_0$ | Tech distance distribution is the same between the categories of Connecting ratio | 0.103 | Retain $H37_0$ |
| $H38_0$ | Degree distribution is the same between the categories of Connecting ratio | 0.253 | Retain $H38_0$ |
| $H39_0$ | Betweenness links distribution is the same between the categories of Connecting ratio | 0.682 | Retain $H39_0$ |
| $H40_0$ | 1. Closeness links distribution is the same between the categories of Connecting ratio | 2. 0.091 | 3. Retain $H40_0$ |

*Reject $H_0$ if p-value < 0.05

Based on the hypothesis listed in the table above (table 9), it is possible identify five main findings:

**Finding 1:** The number of firms has no statistical influence on the number of links, R&D investment, profit, output, technological distance or centrality measures.

**Finding 2:** The number of iterations has no statistical influence on the number of links, R&D investment, profit, output, technological distance or centrality measures.

**Finding 3:** The parameter $a$ has no statistical influence on the number of links, R&D investment, profit, output, technological distance and centrality measures.

**Finding 4:** The parameter $\gamma$ influences the number of links, R&D investment, output, technological distance and centrality measures.

**Finding 5:** The connecting ratio influences the number of links, R&D investment, profit, output, technological distance and centrality measures.

Therefore, we may conclude that our results are not impacted by most of the parameters we have assumed (number of firms, number of iterations and $a$), while being influenced by $\gamma$ and the connecting ratio.

# Chapter 5 - Conclusions and future developments

It is generally recognised that R&D activities have some public good features, as firms cannot fully appropriate the returns of their R&D investments, due to the existence of R&D spillovers. R&D cooperation allows firms to internalise the benefits of research, as well as to capture the economies of scale or complementarities in R&D, and potential beneficial effects coming from firms' coordination of research activities and the diffusion of know-how and R&D output among cooperating firms. Against these advantages is the fear that the participating firms may free-ride on other firms, as well as the possibility of reduction of competition in the product market, which would result in a welfare loss.

A critical issue facing R&D cooperation networks is the selection of a partner among many possible ones, with different technological characteristics. This research intended to develop a matching model of R&D cooperation to study the formation of knowledge alliance decisions based on the technological fit of potential partners. Particularly, we intended to analyse if firms' technological characteristics (e.g. technological distance, R&D investment) influence the network formation.

We started by developing a matching model of R&D cooperation in which firms are endowed with knowledge variables and decide to create, maintain or delete links based on the firm's profit. After an analytical resolution of the model with two homogeneous firms and when comparing competition in R&D with R&D cooperation, our results showed that in a cooperative scenario, each firm has higher R&D and output levels. The intuition is rather simple: when R&D runs cooperatively, R&D equilibrium output increases with the degree of information sharing between firms, leading to a reduction in the productions costs and turning the increase in the output produced more appealing. This result is also evidenced in seminal works such as d'Aspremont and Jacquemin (1988) or Kamien et al. (1992).

We then extended previous analysis by considering $n$ heterogeneous firms and $t$ periods. To further analyse the model, an agent based model was built allowing us to explore the evolution of firms' alliances for R&D purposes. Our results showed that a firm's R&D investment is positively correlated with both profit and output. As it was expected, the highest the profit of a firm, the highest investment the firm might do in R&D activity and vice-versa, the highest the R&D investment, the highest the cost reduction and, consequently, the highest profit; when investing in R&D activities, the cost of production reduces (making more appealing to produce more output).

In addition, our results revealed that there is a positive correlation between firms' technological distance and the number of links formed. As a result, centrality is also positively correlated with the technological distance.

We also observed that there is a positive correlation between *R&D investment* and the centrality (*Degree, Betweenness* or *Closeness)*. This result is rather interesting, revealing that investment in R&D activities influences the position of a firm in the network. This could be explained through the positive correlation between the *R&D investment* and the *number of links*. In addition, firms' *technological distance* is positively correlated with R&D investment.

This dissertation doesn't aims at analysing the network stability or its diameters since that subject was study by many scholars. Instead, it studies a firm's position in a network. Nevertheless, some of these conclusions are in accordance with the results obtained by Cowan *et al.* (2009), Carayol and Roux (2009) and Campo *et al.* (2013).

Even though we use computer simulation to produce the results of a game-theoretic model that describes R&D cooperation among firms, this approach is flawed due to the difficulty of predicting, monitoring and controlling a network. For that reason, we also test our results using the Kruskal-Wallis test in order to analyse the impact of the independent variables in the dependent variables and we concluded that our results are not impacted by most of the parameters we have assumed (number of firms, number of iterations and $a$), while being influenced by $\gamma$ and the connecting ratio.

In spite of the huge effort devoted to this dissertation, this research has other imperfections, such as: i) the R&D spillover is related with technological distance, but not with geographical distance. Despite several researchers subject the R&D spillover to physical distance between firms (others, such as Campos *et al.* (2013) consider both geographical and technological distances), some scholars claim that the development of IT reduce the relevance of physical distance for knowledge transmission; ii) the model assumes that there is complete and perfect information, in particular, every firm knows other firms' knowledge stock even before joining the network. In many real R&D cooperation networks, firms to not reveal to their potential partners their knowledge stock before joining the network. In our model, we avoid incomplete information because that would introduce excessive complexity; iii) for simplicity, we assumed a homogeneous product, but firms may decide to invest in R&D to differentiate their products;

iv) the simulation could include more firms, more periods and run much more times, but as our results showed, that would not introduce significant changes in the results.

This research could be improved by comparing simulated results with real data: in real life, is the selection of R&D cooperation partners driven by technological variables? Also, it might be interesting for a future research to use a centralised procedure to produce R&D networks and compare results with a decentralised one. In this case, we would like to replicate real situations in which an entity (e.g. a business association, an University) has a central role in selecting the partners for R&D purposes. Finally, introducing incomplete information about knowledge stock before joining the R&D network could be an interesting topic to explore.

# Appendixes

## Appendix A – Proof of theorem 1 (Gale and Sotomayor, 1985)

$\mu$ is stable under falsified preferences which we will denote by P'. Further, $\mu$ is the only stable matching for (M, W; P'), for any other matching would leave some $w$ in $\mu(M)$ unmatched, which is not possible. Hence $\mu$ is the M-optimal matching for (M, W; P'). To see that P' is an equilibrium point, suppose some $w$ now changes her preference list leading to a new M-optimal matching $\mu$' which gives her a mate $m' = \mu'(w)$ whom she prefers to $\mu(w)$ under true preference. Then $m'$ must have been rnatched by $\mu$ to some $w'$, for if not $(m', w)$ would have blocked $\mu$ in (M, W; P). But then $w'$ is self-matched under $\mu'$ since $m$' was the only man on her P'-Iist. This means $m$' prefers $w$ to $w$', but if this were so, again $(m', w)$ would have blocked $\mu$, a contradiction.

## Appendix B – Pseudo-code of the Algorithm

```
Initial values and Key parameters
Initialise links between firms (t=0)
Compute technological distance
Compute quadratic spillover function
Compute production cost
Repeat for t=1 to t=10 {Cycle for network evolution)
   Repeat for all firms (i in 1:n) {Cycle for network formation)
      If firms i and j are linked
             Compute firm i's profit without firm j in the network
             Compute firm j's profit without firm i in the network
             Decide to delete or not the link
      If firms i and j are not linked
             Compute firm i's profit with firm j in the network
             Compute firm j's profit with firm i in the network
             Decide to create or not the link
   R&D output update
   Quantity update
   Knowledge stock update
   End cycle of network formation
Graphical representation of the network
Compute network density
```

```
Compute closeness centrality
Compute betweenness centrality
End cycle of network evolution
```

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# Annexes

## Annex 1 – Summary of the main characteristics of articles on R&D cooperation

| Characteristic | Author(s) | Participants | Stages | Cournot / Bertrand | Product | Objective | Main conclusions |
|---|---|---|---|---|---|---|---|
| R&D cooperation models with complete information and exogenous spillovers | Katz (1986) | n | 4 | Cournot | Homogeneous | Analyse the effects of cooperative research. | Firms that share R&D costs have more incentives do develop R&D and firms who share R&D output are more efficient on R&D. |
| | d'Aspremont and Jacquemin (1988) | 2 | 2 | Cournot | Homogeneous | Compare cooperative and non-cooperative R&D level in duopoly with spillovers. | Firms that cooperate in R&D and are rivals in the product market can achieve higher levels of R&D and can increase the quantity produced for large spillovers ($\beta$>0.5). |
| | Kamien *et al*. (1992) | n | 2 | Both | Differentiated | Analyse the effects of R&D cartelization and RJV on firms that engage in either Cournot or Bertrand competition in their product market. | A RJV that cooperates in R&D yields the highest consumer plus producer surplus under Cournot competition and, in most cases, under Bertrand competition. |
| | Suzumura (1992) | n | 2 | Cournot | Homogeneous | Examine the positive and normative effects of cooperative R&D in comparison with non-cooperative R&D, socially first-best R&D and socially second-best R&D. | In the presence of sufficiently large R&D spillovers neither non-cooperative nor cooperative equilibria achieve even second-best R&D levels. In the absence of spillovers effects while the cooperative R&D level remains socially insufficient the non-cooperative level may overshoot first and second best levels of R&D. |
| | Poyago-Theotoky (1995) | n | 2 | Cournot | Homogeneous | Analyse an oligopoly model with information spillovers. | Depending on the magnitude of the spillover, the market may not provide enough incentives for the optimum degree of cooperation to take place. |
| | Matsumura *et al*. (2013) | 2 | 2 | Cournot | Homogeneous | Study the relation between the degree of competitiveness faced in a market by firms and their R&D expenditure. | When the duopoly market is not particularly competitive and when it is highly competitive, R&D activities are intensified. |

| Characteristic | Author(s) | Participants | Stages | Cournot / Bertrand | Product | Objective | Main conclusions |
|---|---|---|---|---|---|---|---|
| R&D cooperation models with complete information and endogenous R&D spillovers | Katsoulacos and Ulph (1998) | 2 | 3 | --- | --- | Examine the effects of RJVs innovative performance in the case where R&D spillovers are endogenously chosen. | When non-cooperation achieves maximal spillovers so does an RJV, whereas minimal non-cooperative spillovers imply partial – but not necessarily maximal – spillovers by an RJV. |
| | Kultti and Takalo (1998) | 2 | 3 | Cournot | Homogeneous | Study if firms have incentives to exchange information taking into account the spillovers. | R&D spillovers can be endogenised in a sense that even without spillovers firms have an incentive to exchange the R&D information after the investment costs are sunk. |
| | Poyago-Theotoky (1999) | 2 | 3 | Cournot | Homogeneous | Analyse a non-tournament model of R&D where firms are engaged in cost-reducing innovation. | When spillovers of information are treated as endogenous firms never disclose any of their information when choosing their R&D non-cooperatively. Under cooperative R&D, firms will always choose to fully share their information. |
| R&D cooperation models with complete information and endogenous size of the cooperative research identity | Combs (1993) | n | 3 | Cournot | Homogeneous | Investigates the effect of stochastic returns to research investments on actual and effective R&D in a model that endogenises the size of the cooperative research identity. | Cooperation tends to increase the underlying probability of research success; total surplus generally increases as more firms cooperate, and cooperation in equilibrium never exceeds the one that maximises total surplus. |

| Characteristic | Author(s) | Participants | Stages | Cournot / Bertrand | Product | Objective | Main conclusions |
|---|---|---|---|---|---|---|---|
| R&D cooperation models with specificities or asymmetries in R&D spillovers | Vonortas (1994) | 2 | 3 | Cournot | Homogeneous | Investigate the effects of inter-firm collaboration in generic research on firms' incentives to undertake both generic R&D and on their market performance. | Joint ventures that simply allow members to coordinate their actions in pre-competitive research can restore firms' incentives for both pre-competitive R&D in the presence of high knowledge spillovers and poor opportunities for innovation. |
| | Steurs (1995) | 4 (2 in each industries) | 2 | Cournot | Homogeneous | Analyse the different impacts of intra- and inter-industry R&D spillovers. | R&D agreements that cut across industries may be more socially beneficial than cooperatives whose membership comes from a single industry. |
| | Amir and Wooders (1999) | 2 | 2 | Cournot | Homogeneous | Analyse the effects of one-way spillovers on market shares, industry price, welfare and R&D cooperation. | A joint lab always improves consumer welfare; it yields higher profits, cost reduction, and social welfare only under extra assumptions, beyond those required with multidirectional spillovers. |
| R&D cooperation models with absorptive capacity | Kamien and Zang (2000) | 2 | 3 | Cournot | Homogeneous | Representation of a firm's 'effective' R&D effort level that reflects how both its R&D approach and R&D budget influences its ability to realize spillovers from other firms' R&D activity ('absorptive capacity'). | When firms cooperate in the setting of their R&D budgets, i.e. form a RJV, they choose identical broad R&D approaches. On the other hand, if they do not form a RJV, then they choose firm-specific R&D approaches unless there is no danger of exogenous spillovers. |
| | Youssef *et al.* (2011) | 2 | 2 | Cournot | Homogeneous | Analyse the case where firms can invest in both innovative and absorptive R&D to reduce their unit production cost considering spillovers. | The investment in innovative R&D is always higher than in absorptive R&D; the value of the learning parameter has almost no impact on innovative R&D, firms' profits, consumer's surplus and social welfare. |

| Characteristic | Author(s) | Participants | Stages | Cournot / Bertrand | Product | Objective | Main conclusions |
|---|---|---|---|---|---|---|---|
| R&D dynamic cooperation models | Petit and Tolwinski (1999) | 2 | Dynamic model | Cournot | Homogeneous | Answer the questions: (i) is the creation of technological cartels beneficial from a social welfare point of view? and (ii) do firms have private incentives to form such cartels? | Antitrust legislation should be flexible towards technological cooperation since it may produce social benefits and even reduce the incentives for industrial concentration. The private incentives of the firms to form technological cartels may change from case to case. |
| R&D cooperation models with coordination costs | Falvey *et al.* (2013) | n | 2 | Cournot | Homogeneous | How coordination costs for the RJV affect the equilibrium outcomes. | There can be profitable but welfare-reducing RJVs and R&D competition can generate a better outcome depending on the extent of coordination costs. |
| R&D cooperation models considering Public Policy | Leahy and Neary (1997) | n | 2 | Both | Homogeneous | Establish the principles which should govern public intervention in industries where R&D is important. | Strategic behaviour by firms tends to reduce output, R&D, and welfare and so justifies higher subsidies except when R&D spillovers are low and firms' actions are strategic substitutes. |
| R&D cooperation networks | Goyal and Moraga-González (2001) | 2 | 3 | Cournot | Homogeneous | Study the incentives for collaboration between horizontally related firms. | In the absence of firm rivalry, the complete network, where each firm collaborates with all others, is uniquely stable, industry-profit maximizing, and efficient. By contrast, under strong market rivalry the complete network is stable, but intermediate levels of collaboration and asymmetric networks are more attractive from a collective viewpoint. This suggests that competing firms may have excessive incentives to form collaborative links. |
| | Deroian and Gannon (2006) | n | 3 | Cournot | Both | Study rival firms' incentives in quality-improving R&D networks. | R&D efforts decrease with the number of partners, networks of alliances are over-connected as compared to the social optimum and the profit-maximising number of alliances is possibly non-monotonic (decreasing then increasing) with respect to inverse measure of product differentiation. |

| Characteristic | Author(s) | Participants | Stages | Cournot / Bertrand | Product | Objective | Main conclusions |
|---|---|---|---|---|---|---|---|
| R&D cooperation networks | Goyal *et al.* (2008) | n | 2 | Both | Both | Develop a model of R&D collaboration in which individual firms carry out in-house research on core activities and undertake bilateral joint projects on non-core activities with other firms and study the relation between the number of joint projects and investments and profits. | Equilibrium investments in in-house as well as in each joint project are increasing in the number of projects. However, an increase in number of joint projects of all firms lowers collective profits, suggesting the presence of excessive incentives for conducting research. |
| | Zirulia (2011) | n | 2 | Cournot | Homogeneous | Analyse the formation of R&D networks in a setting where spillovers between partners may be imperfect, due to knowledge tacitness, and partner specific, depending on firms' technological specialisations. | Firms have strong incentives to use the network to gain a competitive advantage and create (ex post) asymmetries. |

## Annex 2 – Summary of the main characteristics of articles on matching

| Type | Author(s) | Problem | One sided / two sided or multi sided matching | Number of sets | Centralise / Decentralise | Objective | Main conclusions |
|---|---|---|---|---|---|---|---|
| General | Roth (1982) | Marriage | Two sided matching | 2: men and women | Decentralise | Determine the extent to which matching procedures can be designed in order to give agents the incentive to honestly reveal their preferences, and which ones produce stable matches. | No matching procedure exists that always yields a stable outcome and gives players the incentive to reveal their true preferences, even though procedures exist that accomplish either of these goals separately; matching procedures do exist, however, which always yield stable outcome and which always give all the agents in one of the two disjoint sets of agents the incentive to reveal their true preferences. |
| | Alkan (1988) | Family | Multi sided matching | 3: men, women and children | Decentralise | Analyse if the stability theorem is valid for k-some formations. | Stable matching need not exist in societies where threesomes are to form. The stability theorem breaks down for k-some formations for all $k \geq 3$, even when preferences are restricted to be separable. |
| | Zhou (1990) | Generic | One-sided matching | 2: agent and object | Centralise | Discuss issues related to Gale's conjecture. | When there are n objects to be assigned to n agents, for $n \geq 3$, there is no mechanism that satisfies symmetry, Pareto optimality, and strategy-proofness. |
| | Sönmez (1996) | Labor market | Two sided matching (many-to-one) | 2: workers and firms | Decentralise | Search for strategy-proof solutions in the context of (many-to-one) matching problems. | In this model, whenever the firms can hire as many workers as they want (the capacities are unlimited) the stable set is a singleton. There exists a Pareto efficient, individually rational and strategy-proof matching rule if and only if the capacities are unlimited. |

| Type | Author(s) | Problem | One sided / two sided or multi sided matching | Number of sets | Centralise / Decentralise | Objective | Main conclusions |
|---|---|---|---|---|---|---|---|
| General | Jackson and Wolinsky (1996) | Generic | Multi sided matching | N: [1, ..., N ] | Decentralise | Study the stability and efficiency of social and economic networks, when self interested individuals can form or sever links. | There does not always exist a stable network that is efficient this tension persists generally: to assure that there exists a stable network that is efficient, one is forced to allocate resources to nodes that are not responsible for any of the production. |
| | Sherstyuk (1999) | Generic | Multi sided matching | k: $N_1, \ldots, N_k$ | Decentralise | Analyse multisided matching (assignment) games in which players' abilities in a coalition are complementary across types. | Stable matchings are shown to exist when characteristic functions are supermodular, i.e., agents' abilities to contribute to the value of a coalition are complementary across types. |
| | Niederle and Roth (2003) | Entry-level market for American gastroenterologists | Two sided matching | 2: gastroenterologists and hospitals | Both | Investigate the effect of a centralised clearinghouse on the market | The clearinghouse used in gastroenterologists entry-level labour market not only coordinates the timing of appointments but also increases the scope of the market, compared to a decentralised market with early appointments. |
| | Ehlers and Massó (2007) | Entry-level medical markets | Two sided matching | 2: students and hospitals | Centralise | Study ordinal Bayesian Nash equilibria of stable mechanisms in centralised matching markets under incomplete information. | Truth telling is an ordinal Bayesian Nash equilibrium of the revelation game induced by a common belief and a stable mechanism if and only if all the profiles in the support of the common belief have singleton cores. |
| | Femenia *et al*. (2011) | Quota restriction | Multi sided matching | 3: workers of type I, workers of type II and an institution | Decentralise between workers; centralise between institution and pair of workers | Develop a matching market in which an institution has to hire a set of pairs of complementary workers, and has a quota that is the maximum number of candidates pair positions to be filled. | In the unrestricted institution preferences domain, the set of stable solution may be empty. |

| Type | Author(s) | Problem | One sided / two sided or multi sided matching | Number of sets | Centralise / Decentralise | Objective | Main conclusions |
|---|---|---|---|---|---|---|---|
| R&D | Goyal and Moraga-González (2001) | R&D networks | Two sided matching | 2: firms and rivals | Decentralise | Study the incentives for collaboration between horizontally related firms. | In the absence of firm rivalry, the complete network, where each firm collaborates with all others, is uniquely stable, industry-profit maximizing, and efficient. By contrast, under strong market rivalry the complete network is stable, but intermediate levels of collaboration and asymmetric networks are more attractive from a collective viewpoint. This suggests that competing firms may have excessive incentives to form collaborative links. |
| | Miotti and Schwald (2003) | Cooperative R&D | Two sided matching | 2: firms and partners (suppliers, clients, rivals, public institutions or foreign partners) | Decentralise | Develop an integrated framework to examine the determinants of the choice of partners with which firms cooperate on R&D. | The choice of partners is dictated by the complementary resources which the latter command. |
| | Li-ping (2006) | University-industry cooperation | Two sided matching | 2: university and firms | Decentralise | Analyse knowledge transfer process in the context of university-industry cooperation, establishing a five-stage knowledge transfer process model: searching stage, matching stage, learning stage, adaptation stage and integration stage. | The influential factors, which affect university-to-industry effective knowledge transfer, are: university knowledge factor, firm factors, interactive factor, knowledge characteristics and context distance. |

| Type | Author(s) | Problem | One sided / two sided or multi sided matching | Number of sets | Centralise / Decentralise | Objective | Main conclusions |
|---|---|---|---|---|---|---|---|
| R&D | Cowan *et al*. (2009) | Alliance formation and joint innovation | Two sided matching | 2: firms and rivals | Decentralise | Develop a model of R&D networks in which firms seek to innovate, combining theirs and their partners' knowledge in order to produce new knowledge. | Firms are randomly endowed with knowledge elements and base their alliance decisions purely on the technological fit of potential partners, ignoring social capital considerations and indirect benefits on the network. This is sufficient to generate equilibrium networks with the small world properties of observed alliance networks, namely short pairwise distances and local clustering. The equilibrium networks are more clustered than "comparable" random graphs, while they have similar characteristic path length. |
| | Carayol and Roux (2009) | Knowledge flows | Two sided matching | 2: firms and rivals | Decentralise | Demonstrate that the strategic approach to link formation can generate networks that share some of the main structural properties of most real social networks. | For intermediate levels of knowledge transferability, clustering occurs in geographical space and a few agents sustain distant connections. Such networks exhibit the small world property (high clustering and short average relational distances). When the costs of link formation are normally distributed across agents, asymmetric degree distributions are also obtained. |
| | Santamaria and Surroca (2011) | Goals and Impacts of R&D collaboration | Two sided matching | 2: firms with partners (suppliers, clients, rivals, public institutions or foreign partners) | Decentralise | Analyse the fit, or match, between technological goals that lead a firm to choose a particular technological partner and the impact of that partner on the firm's innovation performance. | The motivation in forming alliances with vertical partners is to exploit existing competences; the main driver behind the selection of institutional partnerships is the exploration of new ideas; horizontal collaboration is motivated by the desire to carry out pre-competitive research. |

| Type | Author(s) | Problem | One sided / two sided or multi sided matching | Number of sets | Centralise / Decentralise | Objective | Main conclusions |
|---|---|---|---|---|---|---|---|
| R&D | Campos *et al*. (2013) | Strategies of collaborative networks of R&D | Two sided matching | 2: firms with partners (suppliers, consumers or rivals) | Decentralise | Analyse the evolving dynamics of different strategies of collaborative networks that emerge from the creation and diffusion of knowledge. | Profit is associated with higher stock of knowledge and with smaller network diameter. In addition, concentration strategies are more profitable and more efficient in transmitting knowledge trough the network. |

# Annex 3 – Simulation results for different key parameters

| Key parameters | | | | | Results (last iteration) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Number firms** | **Iterations** | **a** | **γ** | **Connecting ratio** | **Number links** | $\bar{\pi}$ | $\bar{q}$ | $\bar{x}$ | $\bar{d}$ | **Statistical measures** | | |
| | | | | | | | | | | **Degree** | **Betweenness** | **Closeness** |
| 10 | 5 | 3000 | 0.5 | 40% | 6 | 19,479 | 273 | 60 | 0.14 | 1.2 | 1.6 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 40% | 6 | 19,479 | 273 | 60 | 0.14 | 1.2 | 1.6 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 40% | 7 | 19,435 | 273 | 60 | 0.14 | 1.4 | 1.6 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 40% | 6 | 19,479 | 273 | 60 | 0.14 | 1.2 | 1.6 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 40% | 6 | 19,479 | 273 | 60 | 0.14 | 1.2 | 1.6 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 40% | 6 | 19,479 | 273 | 60 | 0.14 | 1.2 | 1.6 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 40% | 7 | 19,435 | 273 | 60 | 0.14 | 1.4 | 1.6 | 0.02 |
| 5 | 5 | 3000 | 0.5 | 40% | 2 | 369,889 | 276 | 83 | 0.33 | 0.8 | 0.2 | 0.07 |
| 5 | 5 | 3000 | 0.5 | 40% | 2 | 369,911 | 276 | 83 | 0.33 | 0.8 | 0.2 | 0.07 |
| 5 | 5 | 3000 | 0.5 | 40% | 4 | 369,901 | 276 | 83 | 0.33 | 1.6 | 1.6 | 0.15 |
| 5 | 5 | 3000 | 0.5 | 40% | 2 | 369,926 | 154 | 44 | 0.33 | 0.8 | 0.2 | 0.07 |
| 5 | 5 | 3000 | 0.5 | 40% | 2 | 369,904 | 154 | 44 | 0.33 | 0.8 | 0 | 0.06 |
| 5 | 5 | 3000 | 0.5 | 40% | 1 | 371,698 | 154 | 46 | 0.33 | 0.4 | 0 | 0.06 |
| 5 | 5 | 3000 | 0.5 | 40% | 1 | 371,708 | 154 | 46 | 0.33 | 0.4 | 0 | 0.06 |
| 7 | 5 | 3000 | 0.5 | 40% | 10 | 262,634 | 280 | 85 | 0.28 | 2.9 | 0.7 | 0.07 |
| 7 | 5 | 3000 | 0.5 | 40% | 10 | 262,639 | 280 | 85 | 0.28 | 2.9 | 2.0 | 0.10 |
| 7 | 5 | 3000 | 0.5 | 40% | 8 | 262,643 | 280 | 85 | 0.28 | 2.3 | 1.1 | 0.06 |
| 7 | 5 | 3000 | 0.5 | 40% | 10 | 262,636 | 280 | 85 | 0.28 | 2.9 | 0.7 | 0.07 |
| 7 | 5 | 3000 | 0.5 | 40% | 9 | 262,643 | 280 | 85 | 0.28 | 2.6 | 2.3 | 0.10 |
| 7 | 5 | 3000 | 0.5 | 40% | 10 | 262,642 | 280 | 85 | 0.28 | 2.9 | 0.7 | 0.07 |
| 7 | 5 | 3000 | 0.5 | 40% | 7 | 262,636 | 280 | 85 | 0.28 | 2.0 | 1.3 | 0.06 |
| 12 | 5 | 3000 | 0.5 | 40% | 20 | -113,558 | 267 | 0 | 0.35 | 3.3 | 4.2 | 0.05 |
| 12 | 5 | 3000 | 0.5 | 40% | 21 | -114,039 | 267 | 0 | 0.35 | 3.5 | 4.1 | 0.05 |
| 12 | 5 | 3000 | 0.5 | 40% | 20 | -112,613 | 266 | 0 | 0.35 | 3.3 | 4.2 | 0.05 |
| 12 | 5 | 3000 | 0.5 | 40% | 20 | -115,296 | 267 | 0 | 0.35 | 3.3 | 4.2 | 0.05 |
| 12 | 5 | 3000 | 0.5 | 40% | 21 | -113,947 | 267 | 0 | 0.35 | 3.5 | 4.1 | 0.05 |
| 12 | 5 | 3000 | 0.5 | 40% | 21 | -110,397 | 266 | 0 | 0.35 | 3.5 | 4.1 | 0.05 |
| 12 | 5 | 3000 | 0.5 | 40% | 20 | -114,328 | 267 | 0 | 0.35 | 3.3 | 4.2 | 0.05 |
| 15 | 5 | 3000 | 0.5 | 40% | 40 | -358,725 | 273 | 0 | 0.27 | 5.3 | 4.3 | 0.05 |
| 15 | 5 | 3000 | 0.5 | 40% | 41 | -356,929 | 272 | 0 | 0.27 | 5.5 | 4.3 | 0.05 |
| 15 | 5 | 3000 | 0.5 | 40% | 39 | -359,128 | 273 | 0 | 0.27 | 5.2 | 4.4 | 0.05 |
| 15 | 5 | 3000 | 0.5 | 40% | 39 | -359,495 | 273 | 0 | 0.27 | 5.2 | 4.4 | 0.05 |
| 15 | 5 | 3000 | 0.5 | 40% | 38 | -359,368 | 273 | 0 | 0.27 | 5.1 | 4.5 | 0.04 |
| 15 | 5 | 3000 | 0.5 | 40% | 39 | -360,698 | 273 | 0 | 0.27 | 5.2 | 4.4 | 0.05 |
| 15 | 5 | 3000 | 0.5 | 40% | 40 | -358,064 | 273 | 0 | 0.27 | 5.3 | 4.3 | 0.05 |
| 10 | 1 | 3000 | 0.5 | 40% | 15 | 73,555 | 268 | 87 | 0.37 | 3.0 | 4.3 | 0.06 |
| 10 | 1 | 3000 | 0.5 | 40% | 10 | 73,721 | 269 | 87 | 0.37 | 2.0 | 5.5 | 0.03 |
| 10 | 1 | 3000 | 0.5 | 40% | 15 | 73,734 | 269 | 87 | 0.37 | 3.0 | 3.6 | 0.06 |

| Key parameters | | | | | Results (last iteration) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number firms | Iterations | a | γ | Connecting ratio | Number links | $\bar{\pi}$ | $\bar{q}$ | $\bar{x}$ | $\bar{d}$ | Statistical measures | | |
| | | | | | | | | | | Degree | Betweenness | Closeness |
| 10 | 1 | 3000 | 0.5 | 40% | 10 | 73,861 | 269 | 87 | 0.37 | 2.0 | 3.6 | 0.02 |
| 10 | 1 | 3000 | 0.5 | 40% | 9 | 73,617 | 269 | 87 | 0.37 | 1.8 | 5.4 | 0.03 |
| 10 | 1 | 3000 | 0.5 | 40% | 12 | 73,631 | 268 | 87 | 0.37 | 2.4 | 5.9 | 0.05 |
| 10 | 1 | 3000 | 0.5 | 40% | 12 | 73,814 | 269 | 87 | 0.37 | 2.4 | 4.2 | 0.03 |
| 10 | 3 | 3000 | 0.5 | 40% | 12 | 23,823 | 281 | 87 | 0.25 | 2.4 | 4.5 | 0.03 |
| 10 | 3 | 3000 | 0.5 | 40% | 12 | 23,843 | 281 | 87 | 0.25 | 2.4 | 5.8 | 0.05 |
| 10 | 3 | 3000 | 0.5 | 40% | 15 | 23,863 | 281 | 87 | 0.25 | 3.0 | 3.7 | 0.06 |
| 10 | 3 | 3000 | 0.5 | 40% | 11 | 23,803 | 281 | 87 | 0.25 | 2.2 | 5.8 | 0.05 |
| 10 | 3 | 3000 | 0.5 | 40% | 13 | 23,861 | 281 | 87 | 0.25 | 2.6 | 4.7 | 0.06 |
| 10 | 3 | 3000 | 0.5 | 40% | 9 | 23,798 | 281 | 87 | 0.25 | 1.8 | 4.8 | 0.03 |
| 10 | 3 | 3000 | 0.5 | 40% | 13 | 23,808 | 281 | 87 | 0.25 | 2.6 | 4.9 | 0.05 |
| 10 | 8 | 3000 | 0.5 | 40% | 4 | 30,808 | 271 | 59 | 0.12 | 0.8 | 0.4 | 0.01 |
| 10 | 8 | 3000 | 0.5 | 40% | 6 | 30,814 | 271 | 59 | 0.12 | 1.2 | 3.5 | 0.02 |
| 10 | 8 | 3000 | 0.5 | 40% | 5 | 30,817 | 271 | 59 | 0.12 | 1.0 | 0.2 | 0.01 |
| 10 | 8 | 3000 | 0.5 | 40% | 6 | 30,817 | 271 | 59 | 0.12 | 1.2 | 0.3 | 0.01 |
| 10 | 8 | 3000 | 0.5 | 40% | 4 | 30,815 | 271 | 59 | 0.12 | 0.8 | 1.0 | 0.01 |
| 10 | 8 | 3000 | 0.5 | 40% | 3 | 30,825 | 271 | 59 | 0.12 | 0.6 | 0.4 | 0.01 |
| 10 | 8 | 3000 | 0.5 | 40% | 5 | 30,824 | 271 | 59 | 0.12 | 1.0 | 0.2 | 0.01 |
| 10 | 10 | 3000 | 0.5 | 40% | 3 | 31,799 | 271 | 58 | 0.13 | 0.6 | 0.0 | 0.01 |
| 10 | 10 | 3000 | 0.5 | 40% | 5 | 31,799 | 271 | 58 | 0.13 | 1.0 | 1.0 | 0.01 |
| 10 | 10 | 3000 | 0.5 | 40% | 4 | 31,798 | 271 | 58 | 0.13 | 0.8 | 0.4 | 0.01 |
| 10 | 10 | 3000 | 0.5 | 40% | 4 | 31,794 | 271 | 58 | 0.13 | 0.8 | 0.8 | 0.01 |
| 10 | 10 | 3000 | 0.5 | 40% | 6 | 31,808 | 271 | 58 | 0.13 | 1.2 | 0.6 | 0.01 |
| 10 | 10 | 3000 | 0.5 | 40% | 6 | 31,799 | 271 | 58 | 0.13 | 1.2 | 0.6 | 0.01 |
| 10 | 10 | 3000 | 0.5 | 40% | 2 | 31,802 | 271 | 58 | 0.13 | 0.4 | 0.0 | 0.01 |
| 10 | 5 | 2500 | 0.5 | 40% | 4 | 3,026 | 227 | 25 | 0.14 | 0.8 | 0.1 | 0.01 |
| 10 | 5 | 2500 | 0.5 | 40% | 5 | 3,011 | 227 | 25 | 0.14 | 1.0 | 0.2 | 0.01 |
| 10 | 5 | 2500 | 0.5 | 40% | 3 | 3,059 | 227 | 25 | 0.14 | 0.6 | 0.1 | 0.01 |
| 10 | 5 | 2500 | 0.5 | 40% | 7 | 2,978 | 228 | 29 | 0.14 | 1.4 | 0.7 | 0.02 |
| 10 | 5 | 2500 | 0.5 | 40% | 5 | 3,004 | 227 | 25 | 0.14 | 1.0 | 0.2 | 0.01 |
| 10 | 5 | 2500 | 0.5 | 40% | 4 | 3,009 | 227 | 25 | 0.14 | 0.8 | 0.1 | 0.01 |
| 10 | 5 | 2500 | 0.5 | 40% | 4 | 2,984 | 227 | 25 | 0.14 | 0.8 | 0.1 | 0.01 |
| 10 | 5 | 2750 | 0.5 | 40% | 4 | 13,997 | 249 | 55 | 0.14 | 0.8 | 0.1 | 0.01 |
| 10 | 5 | 2750 | 0.5 | 40% | 4 | 14,004 | 249 | 55 | 0.15 | 0.8 | 0.4 | 0.01 |
| 10 | 5 | 2750 | 0.5 | 40% | 6 | 13,977 | 249 | 55 | 0.14 | 1.2 | 1.2 | 0.02 |
| 10 | 5 | 2750 | 0.5 | 40% | 5 | 13,990 | 249 | 55 | 0.14 | 1.0 | 1.0 | 0.01 |
| 10 | 5 | 2750 | 0.5 | 40% | 3 | 13,985 | 249 | 55 | 0.15 | 0.6 | 0.0 | 0.01 |
| 10 | 5 | 2750 | 0.5 | 40% | 5 | 14,021 | 249 | 55 | 0.14 | 1.0 | 0.5 | 0.01 |
| 10 | 5 | 2750 | 0.5 | 40% | 6 | 14,051 | 249 | 55 | 0.15 | 1.2 | 1.6 | 0.02 |
| 10 | 5 | 3250 | 0.5 | 40% | 9 | 27,081 | 299 | 77 | 0.15 | 1.8 | 1.7 | 0.02 |
| 10 | 5 | 3250 | 0.5 | 40% | 7 | 27,172 | 299 | 77 | 0.15 | 1.4 | 1.8 | 0.02 |

| Key parameters | | | | | Results (last iteration) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number firms | Iterations | a | γ | Connecting ratio | Number links | $\bar{\pi}$ | $\bar{q}$ | $\bar{x}$ | $\bar{d}$ | Statistical measures | | |
| | | | | | | | | | | Degree | Betweenness | Closeness |
| 10 | 5 | 3250 | 0.5 | 40% | 7 | 27,077 | 299 | 77 | 0.15 | 1.4 | 0.9 | 0.02 |
| 10 | 5 | 3250 | 0.5 | 40% | 12 | 27,128 | 299 | 77 | 0.15 | 2.4 | 6.1 | 0.05 |
| 10 | 5 | 3250 | 0.5 | 40% | 11 | 27,093 | 299 | 77 | 0.15 | 2.2 | 2.2 | 0.03 |
| 10 | 5 | 3250 | 0.5 | 40% | 8 | 27,136 | 299 | 77 | 0.15 | 1.6 | 1.2 | 0.02 |
| 10 | 5 | 3250 | 0.5 | 40% | 8 | 27,116 | 299 | 77 | 0.15 | 1.6 | 1.8 | 0.02 |
| 10 | 5 | 3500 | 0.5 | 40% | 11 | 38,920 | 321 | 80 | 0.15 | 2.2 | 3.8 | 0.04 |
| 10 | 5 | 3500 | 0.5 | 40% | 12 | 38,880 | 321 | 80 | 0.15 | 2.4 | 2.8 | 0.04 |
| 10 | 5 | 3500 | 0.5 | 40% | 12 | 38,904 | 321 | 80 | 0.15 | 2.4 | 3.3 | 0.04 |
| 10 | 5 | 3500 | 0.5 | 40% | 9 | 38,946 | 321 | 80 | 0.15 | 1.8 | 6.8 | 0.03 |
| 10 | 5 | 3500 | 0.5 | 40% | 12 | 38,885 | 321 | 80 | 0.15 | 2.4 | 6.0 | 0.05 |
| 10 | 5 | 3500 | 0.5 | 40% | 15 | 38,941 | 321 | 80 | 0.15 | 3.0 | 3.8 | 0.06 |
| 10 | 5 | 3500 | 0.5 | 40% | 14 | 38,885 | 321 | 80 | 0.15 | 2.8 | 4.0 | 0.06 |
| 10 | 5 | 3000 | 0.3 | 40% | 5 | 20,542 | 273 | 60 | 0.14 | 1.0 | 0.1 | 0.01 |
| 10 | 5 | 3000 | 0.3 | 40% | 7 | 20,580 | 273 | 60 | 0.14 | 1.4 | 1.4 | 0.02 |
| 10 | 5 | 3000 | 0.3 | 40% | 7 | 20,536 | 273 | 60 | 0.14 | 1.4 | 0.4 | 0.01 |
| 10 | 5 | 3000 | 0.3 | 40% | 4 | 20,501 | 273 | 60 | 0.14 | 0.8 | 0.4 | 0.01 |
| 10 | 5 | 3000 | 0.3 | 40% | 9 | 20,561 | 273 | 60 | 0.14 | 1.8 | 1.5 | 0.02 |
| 10 | 5 | 3000 | 0.3 | 40% | 8 | 20,576 | 273 | 60 | 0.14 | 1.6 | 2.1 | 0.02 |
| 10 | 5 | 3000 | 0.3 | 40% | 6 | 20,506 | 273 | 60 | 0.14 | 1.2 | 1.4 | 0.02 |
| 10 | 5 | 3000 | 0.4 | 40% | 7 | 19,990 | 273 | 60 | 0.14 | 1.4 | 1.3 | 0.02 |
| 10 | 5 | 3000 | 0.4 | 40% | 10 | 19,993 | 273 | 60 | 0.14 | 2.0 | 2.9 | 0.03 |
| 10 | 5 | 3000 | 0.4 | 40% | 7 | 19,970 | 273 | 60 | 0.14 | 1.4 | 0.7 | 0.02 |
| 10 | 5 | 3000 | 0.4 | 40% | 4 | 19,977 | 273 | 60 | 0.14 | 0.8 | 0.1 | 0.01 |
| 10 | 5 | 3000 | 0.4 | 40% | 6 | 19,997 | 273 | 60 | 0.14 | 1.2 | 0.2 | 0.01 |
| 10 | 5 | 3000 | 0.4 | 40% | 8 | 19,978 | 273 | 60 | 0.14 | 1.2 | 2.0 | 0.02 |
| 10 | 5 | 3000 | 0.4 | 40% | 6 | 20,001 | 273 | 60 | 0.14 | 1.6 | 1.0 | 0.02 |
| 10 | 5 | 3000 | 0.6 | 40% | 4 | 18,925 | 273 | 60 | 0.14 | 0.8 | 0.1 | 0.01 |
| 10 | 5 | 3000 | 0.6 | 40% | 6 | 18,871 | 273 | 60 | 0.14 | 1.2 | 0.6 | 0.01 |
| 10 | 5 | 3000 | 0.6 | 40% | 7 | 18,915 | 273 | 60 | 0.14 | 1.4 | 2.3 | 0.02 |
| 10 | 5 | 3000 | 0.6 | 40% | 6 | 18,895 | 273 | 60 | 0.14 | 1.2 | 0.5 | 0.01 |
| 10 | 5 | 3000 | 0.6 | 40% | 5 | 18,901 | 273 | 60 | 0.14 | 1 | 0.4 | 0.01 |
| 10 | 5 | 3000 | 0.6 | 40% | 8 | 18,911 | 273 | 60 | 0.14 | 1.6 | 3.2 | 0.03 |
| 10 | 5 | 3000 | 0.6 | 40% | 8 | 18,898 | 273 | 60 | 0.14 | 1.6 | 1.0 | 0.02 |
| 10 | 5 | 3000 | 0.7 | 40% | 4 | 18,377 | 273 | 60 | 0.14 | 0.8 | 0.2 | 0.01 |
| 10 | 5 | 3000 | 0.7 | 40% | 5 | 18,366 | 273 | 60 | 0.14 | 1 | 0.8 | 0.01 |
| 10 | 5 | 3000 | 0.7 | 40% | 9 | 18,326 | 273 | 60 | 0.14 | 1.8 | 0.7 | 0.02 |
| 10 | 5 | 3000 | 0.7 | 40% | 6 | 18,412 | 273 | 60 | 0.14 | 1.2 | 0.7 | 0.01 |
| 10 | 5 | 3000 | 0.7 | 40% | 9 | 18,356 | 273 | 60 | 0.14 | 1.8 | 3 | 0.03 |
| 10 | 5 | 3000 | 0.7 | 40% | 7 | 18,379 | 273 | 60 | 0.14 | 1.4 | 1.2 | 0.02 |
| 10 | 5 | 3000 | 0.7 | 40% | 7 | 18,356 | 273 | 60 | 0.14 | 1.4 | 1.4 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 30% | 5 | 19,462 | 273 | 60 | 0.14 | 1 | 1 | 0.01 |

| Key parameters | | | | | Results (last iteration) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number firms | Iterations | a | γ | Connecting ratio | Number links | $\bar{\pi}$ | $\bar{q}$ | $\bar{x}$ | $\bar{d}$ | Statistical measures | | |
| | | | | | | | | | | Degree | Betweenness | Closeness |
| 10 | 5 | 3000 | 0.5 | 30% | 7 | 19,456 | 273 | 60 | 0.14 | 1.4 | 1.1 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 30% | 6 | 19,465 | 273 | 60 | 0.14 | 1.2 | 2.9 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 30% | 7 | 19,424 | 273 | 60 | 0.14 | 1.4 | 1.4 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 30% | 6 | 19,504 | 273 | 60 | 0.14 | 1.2 | 0.6 | 0.01 |
| 10 | 5 | 3000 | 0.5 | 30% | 8 | 19,431 | 273 | 60 | 0.14 | 1.6 | 1.0 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 30% | 2 | 19,444 | 273 | 60 | 0.14 | 0.4 | 0 | 0.01 |
| 10 | 5 | 3000 | 0.5 | 35% | 6 | 19,452 | 273 | 60 | 0.14 | 1.2 | 1.6 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 35% | 6 | 19,435 | 273 | 60 | 0.14 | 1.2 | 2 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 35% | 6 | 19,465 | 273 | 60 | 0.14 | 1.2 | 0.3 | 0.01 |
| 10 | 5 | 3000 | 0.5 | 35% | 10 | 19,416 | 273 | 60 | 0.14 | 2 | 2.7 | 0.03 |
| 10 | 5 | 3000 | 0.5 | 35% | 4 | 19,469 | 273 | 60 | 0.14 | 0.8 | 0.1 | 0.01 |
| 10 | 5 | 3000 | 0.5 | 35% | 3 | 19,408 | 273 | 60 | 0.14 | 0.6 | 0.1 | 0.01 |
| 10 | 5 | 3000 | 0.5 | 35% | 5 | 19,469 | 273 | 60 | 0.14 | 1 | 1 | 0.01 |
| 10 | 5 | 3000 | 0.5 | 45% | 7 | 19,458 | 273 | 60 | 0.14 | 1.4 | 4.4 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 45% | 7 | 19,459 | 273 | 60 | 0.14 | 1.4 | 0.4 | 0.01 |
| 10 | 5 | 3000 | 0.5 | 45% | 9 | 19,437 | 273 | 60 | 0.14 | 1.8 | 3.8 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 45% | 8 | 19,496 | 273 | 60 | 0.14 | 1.6 | 2.1 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 45% | 6 | 19,494 | 273 | 60 | 0.14 | 1.2 | 1.3 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 45% | 9 | 19,414 | 273 | 60 | 0.14 | 1.8 | 0.7 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 45% | 5 | 19,464 | 273 | 60 | 0.14 | 1 | 0.2 | 0.01 |
| 10 | 5 | 3000 | 0.5 | 50% | 6 | 19,439 | 273 | 60 | 0.14 | 1.2 | 2 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 50% | 7 | 19,458 | 273 | 60 | 0.14 | 1.4 | 1.4 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 50% | 5 | 19,418 | 273 | 60 | 0.14 | 1 | 2 | 0.02 |
| 10 | 5 | 3000 | 0.5 | 50% | 9 | 19,435 | 273 | 60 | 0.14 | 1.8 | 3 | 0.03 |
| 10 | 5 | 3000 | 0.5 | 50% | 5 | 19,439 | 273 | 60 | 0.14 | 1 | 1 | 0.01 |
| 10 | 5 | 3000 | 0.5 | 50% | 5 | 19,477 | 273 | 60 | 0.14 | 1 | 0.8 | 0.01 |
| 10 | 5 | 3000 | 0.5 | 50% | 8 | 19,437 | 273 | 60 | 0.14 | 1.6 | 1 | 0.02 |